Review Notes

EC 4010

Long-Run General Equilibrium Model

Up until now, we have taken the interest rate as given in our standard two period model. Yet in reality, nothing is simply *given* at the aggregate level: everything is endogenous. In a partial equilibrium analysis, we take prices—here, the interest rate—as exogenous. While individual consumers surely take the interest rate as given, at the economy-wide level the interest rate is a market-determined price. And like all prices, this is determined by the interaction of supply and demand. In a general equilibrium model, we solve for demands *and* prices. To start with, I want to analyze what is called the *natural rate of interest.* You can consider this the fundamental or average real rate that should prevail in an economy *on average.* For instance, if you were asked what the real rate will be in 100 years time, the answer would be the natural rate. It's the average rate the prevails when the economy is neither in boom nor recession. Therefore, implicit in what follows is the assumption that the economy is at potential, with output and demand given by *potential output* (a level determined by a long-run growth model such as the Solow model.) For simplicity, I assume there's no uncertainty, no growth, and no risk. There are free, frictionless markets, prices are flexible, and the classical dichotomy holds. And because I am not yet introducing money, think of everything in terms of goods.[1] For example, if I lend you 100 coconuts and you give me 104 back, then $r = 4\%$. Because of this long-run setting, please forget about the Federal Reserve, money, Keynes, recessions, IS-LM, booms and all that short-run material for now. We seek to find the natural rate at any point in time. For the determination of the natural rate, the supply of loanable funds comes from savings, while the demand stems from investment. As a result, the natural rate stems from the interaction of consumption and production decisions—and will ultimately depend of preferences and technology. In the model below, there is a representative agent and firm. I assume households place all of their savings in the financial markets. Investors then borrow these for investment—and the natural rate ensures equilibrium. As such, we are assuming financial intermediation works well: financial markets ensure the stock of savings flows into investment. Rather that saying savings and investment, it's common to talk in terms of bond demand and bond supply. That is, people save by demanding bonds, while firms invest by supplying or issuing bonds. The economy lasts for two periods. But, more generally, we can extend the analysis to arbitrarily many periods. As we already know, the standard Euler equation gives the optimal allocation of consumption across two periods, but this holds for *any* two consecutive periods for *any* arbitrary length of time. So the standard first order conditions pin down the optimal consumption path for any length of time; implicitly this gives us the savings decision. Restricting to two periods just simplifies the analysis. Anyway, lifetime utility is

$$u(C_1) + \beta u(C_2) \qquad u' > 0, \qquad u'' < 0.$$

The budget constraint in the first period is

$$C_1 + B_1 = Y_1.$$

Here, B_1 are bonds. We could also write this as savings and simply assume agents save through purchasing bonds. If B_1 is positive, the consumer demands bonds.

[1] Equivalently you could say that everything is in nominal terms, but prices are normalized to one. We simply get rid of prices, since money is neutral in this model.

This just means the consumer is saving, and $S_1 = B_1$.[2] In the second period, the budget constraint is

$$C_2 = Y_2 + (1+r)B_1,$$

that is, the consumer consumes period 2 income plus the return on the bonds. Manipulating these constraints gives the consolidated or *intertemporal* budget constraint. The standard lifetime income constraint is

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}.$$

So where does the income come from? Well, one obvious candidate is labour supply. But because we are not interested in production here, we won't model the sources of income, Y_1 and Y_2. Both refer to potential output. The consumer takes r as given and maximizes lifetime utility subject to the intertemporal budget constraint. The standard Euler equation is

$$\underbrace{u'(C_1)}_{pain} = \underbrace{\beta(1+r_1)u'(C_2)}_{gain}$$

Now, this is the usual *pain versus gain* condition.[3] At equilibrium the utility loss from giving up a little consumption equals the utility gain. I give up one good and lose $u'(C_1)$. Next period I get $(1+r_1)u'(C_2)$ goods. And since I discount the future by β, the utility gain to saving in period one is $\beta(1+r_2)u'(C_3)$. If this doesn't hold, we can shift around consumption a bit and do better. Implicitly, this pins down the optimal evolution of consumption. (Always bear in mind that *all* first order conditions pin down optimal demands (or demand curves).)

Combining the intertemporal budget constraint with the Euler equations give the optimal *levels* of C_1 and C_2, and in turn the optimal degree of saving in period 1. Yet, from a theoretical standpoint, it's not clear whether savings are increasing or decreasing in the real interest rate. Yet I assume $S'(r) > 0$; that is, savings are increasing in the real rate of return. Empirically, for any given saver, there is is in

[2] In this analysis, supply of savings equals demand for bonds.

[3] If we had more periods, we'd have $u'(C_2) = \beta(1+r_2)u'(C_3)$ and so on.

fact little response of savings to interest rate changes; i.e., $s'(r) \approx 0$. Note that, in addition to the usual income and substitution effects, there is another effect: *the wealth effect*: when the interest rate rises, the presented discounted value of future income, $Y_1 + \frac{Y_2}{1+r}$, falls and this makes the consumer feel poorer. Then, by the permanent income hypothesis, consumption will fall today. This is another reason why increases in the interest rate tend to raise savings. This wealth effect always acts in the same direction of the substitution effect, thus reinforcing it.

The sensitivity of savings to the change in interest rates depends on the concavity of the utility function and, specifically, the *intertemporal elasticity of substitution*. This parameter determines how much the consumer is willing to shift consumption across periods and depends crucially on the degree of diminishing marginal utility to consumption each period; i.e., the concavity of the utility function. (To see what I'm talking about, content yourself that the reason you don't skip lunch today and have two tomorrow instead is that there's sharp diminishing marginal utility to lunches (making utility very concave). Therefore, your response to a large interest rate—say, give me your lunch today, and Ill give you five lunches tomorrow—is likely small.)

0.0.1 The Representative Firm

Turning now to the representative firm, the firm represents the *borrower* and supply of bonds. The firm is perfectly competitive—a standard assumption in long-run models. For this reason, the firm takes the price of its goods as given (that's why I simply normalize the price to one.) Again, everything is in real terms. The firm's profits derive from the number of items produced—given by production—less the cost of production–the purchase price of capital. Investment takes place in period 1, while production occurs in period 2.[4] There is no labour in the model. The firm starts off with no capital, but in period 1, the firm borrows an amount K (i.e., the investment), and promises to pay back $(1 + r)K$ at the very end of the period 2. The firm purchases investment in period 1, so as to have capital in period 2. For

[4] I'm implicitly assuming income in period 1 is given exogenously by Y_1, but in more realistic settings, it could be determined by labour, or an initial stock of capital.

this reason, K in period 2 derives solely from investment, I, in period 1. The firm takes r as given and maximizes the present discounted value of profits in period 2:

$$\pi = \frac{Af(K) - (1+r)K}{1+r}, \qquad f'' < 0.$$

This is a static problem: there are no dynamics to the firm's choice of investment.

Maximizing with respect to K gives

$$\underbrace{Af'(K)}_{gain} = \underbrace{1+r}_{pain}$$

Note that the above is an equilibrium condition, not a definition. Think of it as a *rule* that dictates the firm's optimal plans. Because $f'' < 0$, investment is decreasing in the interest rate. If the right hand side is high, the left hand side is also high; that is, the marginal product of capital is high, meaning capital demand, and hence investment, is low. According to this condition, therefore, when the interest rate is high, investment is low.[5] This implicitly defines the negatively sloped demand curve for investment, and in turn, demand for loanable funds. Combining this downwardly sloping demand curve with an upward sloping supply curve, $S(r)$, gives the natural rate, r.

0.0.2 General Equilibrium

At the natural rate we have

$$Af'(K^*) = 1 + r^*$$

and

$$u'(C_1^*) = \beta(1 + r^*)u'(C_2^*)$$

In equilibrium, everything is endogenous: the price, r^*, and demands, K^*, C_1^* and C_2^*. Combining the optimality conditions gives the condition:

$$u'(C_1^*) = \beta Af'(K^*)u'(C_2^*)$$

[5]Note that this relationship is not specific to this model; it also holds in more sophisticated model such as Tobin's Q model of investment.

$$\frac{u'(C_1^*)}{\beta u'(C_2^*)} = Af'(K^*) \qquad \Rightarrow \text{MRS=MRT}$$

$$\underbrace{\frac{u'(C_1^*)}{\beta u'(C_2^*)}}_{MRS} = \underbrace{Af'(K^*)}_{MRT}$$

Recall that this is a standard general equilibrium condition. Now this is an old friend from the micro part of EC3010. Optimality dictates that the rate at which you substitute consumption across periods equals the rate at which it's technologically possible. If you think about it, this makes sense. Imagine technology permits you to transfer one good today into 3 tomorrow: this is what the marginal rate of transformation, MRT, tells you; it's what "the world" or God permits you to do. The MRS tells you the rate at which *you* personally *desire* to substitute one unit today for another tomorrow. If it were 2, for instance, Id be happy to give up 1 unit today for 2 tomorrow. But at these figures, the MRT offers a great deal. Technology permits you to give up one today for 3 tomorrow, but you'd be happy to do the exchange at a rate of 1 for 2. It follows that you'll continue making these exchanges until the MRT=MRS (by continuing to make the exchanges, the marginal product of capital *falls* next period, while the MRS rises; the "transfers" end when both are equal.)

0.1 Remarks

Note that the natural rate is *jointly* determined on the production and consumer side. As an example, suppose investment increases, causing the natural rate to rise. The subsequent rise in the natural rate depends on how responsive consumers are to rising interest rates, as we move up along the savings curve. In other words, the attendant rise in equilibrium savings and investment depends on the *intertemporal elasticity of substitution* (IES). As a result of the rise in investment—in the case of an elastic supply of savings—consumption will rise over time. In the extreme case where consumers are completely irresponsive to rising interest rates (i.e., $IES = 0$), the savings line would be vertical—and all the burden of adjustment would fall in the interest rate (i.e., on prices rather than quantities.) That is, consumers save more today and ultimately consume more tomorrow. To take a related example, imagine we have identical two closed economics (A and B), except that investment demand rises in A, but not in B. As a result, consumption growth will be higher in A (as people save more in response to higher interest rates induced by greater investment demand.) Fundamentally, the different consumption behaviours will be due to different investment rates in both countries. Another point to note is that different marginal products of capital across countries do not necessarily reflect technological advantage. To see this, suppose we have two different closed economies—A and B—where investment demand is the same in both, but savings are higher in A. In equilibrium, the marginal product of capital is higher in B. Yet this does not mean B has some technological advantage. Instead, it means that in equilibrium, the relative scarcity of savings in B means lower investment in equilibrium—and so the marginal product of capital will be higher there. Moreover, because parameters such as β lower savings, the high equilibrium marginal product of capital in B would be attributable *fundamentally* to a higher value of β in A. Point is, in a general equilibrium model, prices depend on structural features of the economy.

From the permanent income hypothesis we know that consumption—and hence savings—does not vary *that* much over time. For this reason, most changes in the natural rate stem from changes in the level of investment.

Output is always fixed at potential in this setup. All we are concerned about is the *distribution* of output among government expenditure, consumption, investment and net exports. The interest rate is the key to adjustment. For instance, if consumption increases, then saving falls and the interest rate rises. Investment then falls so we again have aggregate demand equal to potential again. Output is always *fixed* at potential. If one component of demand falls, a fall in interest rates will "crowd in" another part. Similarly, if one component of demand rises, this will "crowd out" another part.[6]

[6]Contrast this with short-run Keynesian models, where a rise in the savings rates induces a recession.

You might wonder, if the stock of savings increases over time as the economy grows, does this mean that the natural rate falls over time? No. As we know from long-run growth models, productivity is the main source of rising living standards. And according to these growth models, a rise in productivity leads to more savings *and* investment (more savings, since you are saving a roughly constant portion of a bigger pie; and more investment since new technologies are continually raising the marginal product of capital.) Empirically, therefore, savings and investment/GDP ratios are roughly constant over time, and the natural rate is fairly stable over time.

In this long-run model, I'm assuming prices are flexible, and so we are always at potential output. Note that is savings equal investment, we have

$$S = I$$

But in our simple economy

$$S = Y^* - C$$

Hence

$$Y^* - C = I \Rightarrow Y^* = C + I,$$

that is, equilibrium in financial markets implies equilibrium in the goods market (i.e., demand, $C + I$, equals supply, Y^*.)

Finally, I'm assuming away the government, but keep in mind that, in reality, government savings constitute an important part of national savings.

0.1.1 Extensions: Large Open Economy

So far we have assumed the only source of funds is household savings. Yet this is plainly unrealistic. Two further sources are government savings and international savings. Suppose capital flows in to a country when interest rates rise. We typically model capital flows, CF, as $CF = f(r - r_f)$ (where $f' > 0$), so when domestic interest rates rise above foreign ones, capital would flow in. Thus this gives another reason why $S'(r) > 0$. Graphically, this would make the savings curve flatter—so a rise in investment would mean interest rates don't rise as much. That is, the rise in international savings attenuates the rise in interest rates caused by an increase in demand for funds. Take a rise in investment, for instance. As interest rates rise, new capital comes in from abroad. That ultimately attenuates the rise in interest rates as a result of higher investment. Because capital flows away from other countries, this dynamic raises interest rates in those countries. Eventually, we would expect all the capital flow to equate risk-adjusted interest rates everyone; this tendency for globalization to equate factor prices is called *factor price equalization.*[7] Yet, a country may be sufficiently large that capital inflows may not be sufficient to lower interest rates that much. By contrast, for a small open economy, the interest rate is given and the supply of savings is perfectly elastic. In this case, increases in investment have *no* effect on domestic interest rates.

With capital mobility, domestic savings need note equal domestic investment. For example if national savings, $S_p + S_g$, lie below investment demand, then the country must import the discrepancy; at the end of the day, that investment must be financed from *somewhere.* And we know, it's not all from national savings; hence, by a process of elimination, it must come from foreign savings. Yet this is not a free lunch; the *net foreign asset* position of the country deteriorates, and the domestic country becomes increasingly foreign-owned. Conversely, suppose in equilibrium that $I < S_p + S_g$; i.e., less than national savings. In this case, there must have been capital outflows from the domestic country.

[7]There is an important caveat here. Due to expected changes in exchange rates, interest rates are rarely equal across countries (even when levels of risk are the same). For example, if interest rates are higher in Britain than Ireland, I still might decide to invest here. Namely, if I invested in Britain and *then* sterling depreciated, my *euro denominated* return—which is what I care about—could conceivably be quite low even in the face of higher sterling-denominated returns in Britain. As a first order approximation, however, the assumption of equal risk-adjusted returns across countries is a reasonable one.

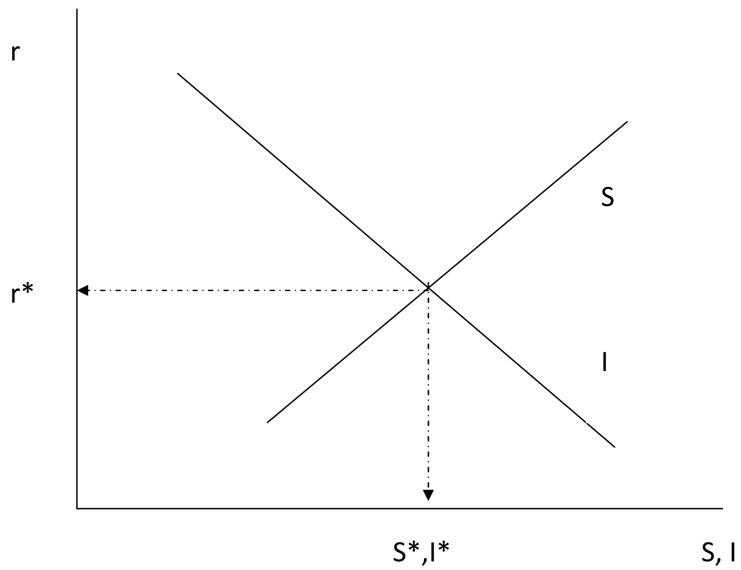

Figure 1: Financial market equilibrium. At equilibrium, savings equal investment. When the interest rate is above the equilibrium rate, savings supply exceeds investment demand. This surplus causes the price of loanable funds—i.e., the interest rate—to fall. As a result, two things happen. First, because saving is becoming less attractive, some savers decide to leave the market. Second, as the cost of funds falls, some investors are seduced into the market. The fall in savings and rise in investment demand causes the initial differential between savings and investment to fall. This process continues until both supply and demand for funds are equal—which occurs at the intersection of the curves. The opposite forces dominate if the economy starts below the equilibrium interest rate. In contrast to the Solow model, we assume these adjustments occur almost instantaneously.

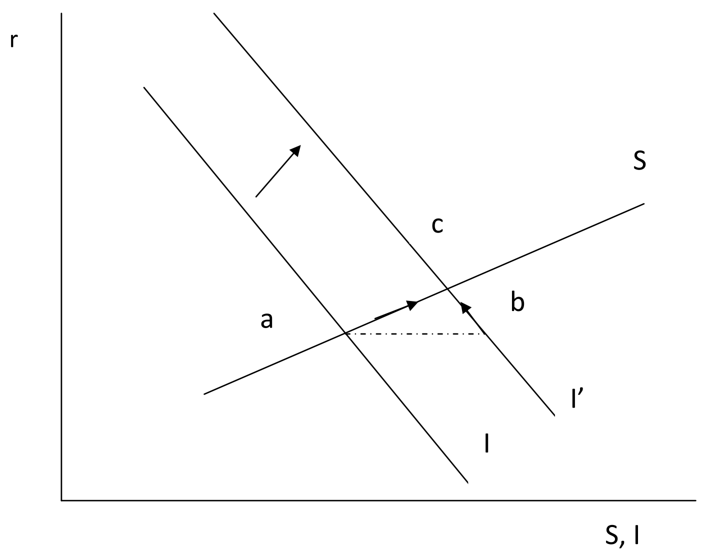

Figure 2: Rise in Productivity, A, with a High Intertemporal Elasticity of Substitution. The rise in productivity raises the marginal product of capital and, in turn, investment demand. In this case, the utility function is not so concave, and savers are relatively responsive to rising interest rates. As a result, savings increases a lot when the interest rate rises. In equilibrium, savings, investment, and the interest rate are higher. The more elastic the savings response, the greater the rise in actual investment in equilibrium. This is because savers readily supply more funds as the interest rate rises, and this increase in supply can finance more investment.

Aside: The Real Exchange Rate

When discussing twin deficits, we noted that a rise in U.S. interest induced a rise in the *real exchange rate*, which in turn caused the current account to deteriorate.[8] To understand what the real exchange rate is, suppose the cost of a good in the U.S. is 100 dollars. Lets call this P^*. Let's say the exchange rate e is 2; that is, one euro purchases 2 dollars. Then in euro terms, the cost of the good is $\frac{100}{2} = \frac{P^*}{e} = 50$. Suppose now the good costs 60 at home; lets call this P. Then the good is relatively expensive domestically. Formally, the relative price of the good at home is

$$\frac{P}{\frac{P^*}{e}} = \frac{eP}{P^*} = \epsilon$$

Because of its importance, the term above is called the *real exchange rate.* When calculating the real exchange rate, we are not really interested in comparing the prices of individual goods; rather, we are concern about the relative prices of *baskets* of goods. Therefore, the prices in the formula refer to *price levels* such as the CPI. The real exchange rate is a true measure of *competitiveness* of a country. If the real exchange rate is high, foreign goods are relatively cheap, and domestic goods are relatively expensive. And you know what this means? Net exports will fall, and hence we often write $CA(\epsilon)$; i.e., the current account is a (decreasing) function of the real exchange rate.

What causes the real exchange rate to change? A fall in ϵ can be attributable to three things a) a rise in P^* or b) a fall in e ;i.e., a currency depreciation, and c) a fall in the domestic price level, P. Note that each of these movements makes us *more competitive.* As such, a fall in ϵ is a good thing. Conversely, a rise in ϵ makes us less competitive vis-à-vis the foreign country.

[8]Essentially, the current account is the trade deficit (but it also incorporates interest payments on foreign debt.)