Debt Sustainability

Key variable is $\frac{D}{Y}$

$$\triangle D = G - T + rD$$

$$\frac{\triangle D}{D} = \frac{G - T}{D} + r$$

Growth of $\frac{D}{Y}$

$$\frac{\triangle \frac{D}{Y}}{\frac{D}{Y}} = \frac{G - T}{D} + r - g$$

$$\triangle \frac{D}{Y} = \frac{G - T}{Y} + (r - g)\frac{D}{Y}$$

For stability, need $\triangle \frac{D}{Y} = 0$

$$\frac{T - G}{Y} = (r - g)\frac{D}{Y}$$

New Keynesian: output demand determined

Demand main source of output fluctuations (e.g., government expenditure)

More generally: an array of rigidities such as inflexible prices. Frictions such as unions.

Today: NK and RBC. Salt/fresh water. Debate about quantitative importance of channels.

Real business cycle theory: variations in supply

Money neutral (no money in model). Empirical relationship not a problem.

Composition and Level of Y_t change

Really, an extension of Ramsey-Cass-Coopmans model (the Solow model with an endogenous savings rate)

Fluctuations represent optimal responses to economic conditions. Fluctuations are Pareto Optimal (follows from perfect competition assumption and First Welfare Theorem.)

RBC: technology/TFP shocks

Shocks are real. Everything in real terms. No nominal variables.

TFP central to growth theory.

Empirically, Solow residual is procylical.

Technology shocks change potential $Y = A_t K_t^{\alpha} L_t^{1-\alpha}$

For now, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t$

A is anything that changes amount for output for given K and L; e.g., inventions, oil, taxation, weather, regulation, bank failures/failures of financial intermediation.

As a result, they change MPK and MPL

No role for demand, output always at potential (and int rate at r_n.) No FED.

Potential varies

Long-run model: recall central role of technology, A.

Capital accumulation important too.

What if technology varies cyclically?

Robinson crusoe example (fish; weather)

Fluctuations are optimal

"Unemployment" voluntary. Intensive/Extensive Margins

First welfare theorem; stabilization policy

Good fit with very basic model

Income/Sub effects

eg permanent change in wage

Temporary rise in wage above trend

Temporary rise in interest rate

Income effects small due to PIH

"Make hay while sun shines"

Fish example

Consequence for saving, investment, and next period's capital

Persistence important for capital accumulation; if not persistent no need to invest

Shocks can't be "too temporary" since consumption *does* rise moderately. So shocks are temporary and persistent.

Propagation mechanisms

Shocks must be temporary and somewhat persistent

First key idea: Intertemporal substitution of labour

Productivity shock raises labour demand.

Rise in wages causes people to "make hay while sun shines"

Rise in wage must be temporary, making the substitution effect small (A permanent rise could cause labour supply to *fall* if income effect was strong enough)

People also increase labour supply in response to interest rate fluctuations

Second key idea

Capital accumulation.

With productivity shock, lifetime wealth goes up.

By PIH, we smooth that over lifetime (since income increase only temporary.) So consumption only goes up a little today.

But output goes up a lot today due to A and L increase.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} = C_t + I_t$$

Savings used for investment

Model predicts procyclical consumption and *highly* variable investment.

Persistent business cycles.

Higher investment this period: implication for next period?

$K_{t+1} = I_t + K_t$

Capital stock is higher next period, thereby causing *persistence*.

Representative Firm and Household

$$E_0 \sum_{t=0}^{t=\infty} \beta^t \left(\log C_t - \frac{l_t^{1+\sigma}}{1+\sigma} \right)$$

$$w_t l + r_t k_t = C_t + i_t$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

No profits in budget constraint (since firms are perfectly competitive)

Combining

$$w_t l_t + r_t k_t = C_t + k_{t+1} - (1-\delta)k_t$$

$$w_t l_t + r_t k_t = C_t + k_{t+1} - k_t + \delta k_t$$

$$w_t l_t + (1 + r_t - \delta)k_t = C_t + k_{t+1}$$

$$\underbrace{w_t l_t + (1 + r_t - \delta)k_t}_{sources} = \underbrace{C_t + k_{t+1}}_{destinations}$$

Tradeoff

Time constraint

Set $\delta = 0$ (or think of r_t as real return *net of depreciation*)

$$u'(C_t) = E_t \beta(1 + r_{t+1})u'(C_{t+1})$$

Trajectory of A is uncertain, but we know its expectation and variance. As a result, r_{t+1} is uncertain; hence we need an expectation sign in Euler equation.

Note that r_{t+1} is int rate in Euler equation; buy today to rent out next period.

Labour optimality condition:

$$w_t u(C_t) = v'(l_t)$$

Implicitly, gives labour supply

$$w_{t+1} u(C_{t+1}) = v'(l_{t+1})$$

$$\lim_{t \to \infty} \beta^t u'(C_t) k_t = 0$$

$\beta = \frac{1}{1+\rho}$.

Like before, w and r are endogenous (a general equilibrium model)

Since $u(C_t) = \log C_t$

$$\frac{w_t}{C_t} = l_t^\sigma$$

$$\frac{w_{t+1}}{C_{t+1}} = l_{t+1}^\sigma$$

$$\frac{1}{C_t} = E_t \beta (1 + r_{t+1}) \frac{1}{C_{t+1}}$$

For simplicity set $\beta(1 + r_t) = 1$ and ignore uncertainty

$$\frac{l_{t+1}^\sigma}{l_t^\sigma} = \frac{w_{t+1}}{w_t}$$

$$\frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t} \right)^{\frac{1}{\sigma}}$$

This represents the *intertemporal substitution of labour.*

More generally, when $\beta(1 + r_t) \neq 1$ and no uncertainty:

$$\frac{l_{t+1}^{\sigma}}{l_t^{\sigma}} = \frac{w_{t+1}}{w_t}\frac{1}{\beta(1 + r_{t+1})}$$

$$\frac{l_{t+1}}{l_t} = \left(\frac{w_{t+1}}{w_t}\frac{1}{\beta(1 + r_{t+1})}\right)^{\frac{1}{\sigma}}$$

Great vacation

Linearity in labour

Important point: degree of intertemporal substitution depends on σ.

RBC'ers must argue σ is relatively low for intertemporal substitution to be important.

The Firm

Price taker, perfect competition

$$\pi = AK^{\alpha}L^{1-\alpha} - wL - rK$$

$$(1-\alpha)AK^{\alpha}L^{-\alpha} - w = 0$$

First order conditions:

$$\frac{\partial \pi}{\partial L} = (1-\alpha)AK^{\alpha}L^{\alpha} - w = 0$$

Implicitly, gives labour demand

$$\frac{\partial \pi}{\partial K} = \alpha AK^{\alpha-1}L^{\alpha} - r = 0$$

Implicitly, gives capital demand.

Solve for L and K in first order conditions above to get labour and capital demands.

Key point: *Labour and Capital demand are both increasing in* A

More importantly, wages and rental rates are increasing in A.

General equilibrium (Combine HH and Firm)

What increases MPK and MPL (like before)

Major point; wages and int rates depend on capital stock and capital changes a lot here

$$\frac{\partial F}{\partial L}L + \frac{\partial F}{\partial K} = wL + rK = F$$

CRS; zero profits.

Labour prod

Solow residual

NK response: Solow residual is not exogenous to cycle

Think of restaurant and labour hoarding

labour hoarding, IRS, varying capital/labour utilization

$Y_t = A(uK)^{\alpha}(uL_t)^{\alpha}$

where u denotes effort (say)