Review Notes

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Chapter 1

Labour Supply

Now we turn to labour supply. Introducing labour supply into the standard two-period model means there are now effectively two goods: consumption and leisure. Here, leisure is effectively a consumption good that the household "consumes." By supplying labour and taking less leisure, the household is implicitly exchanging the leisure good for more regular consumption goods. The *period utility function*—i.e., the utility function at a given point in time—now takes the form:

$$u(c, l) = u(c) - v(l),$$

where c denotes consumption and l denotes labour supply. For the usual reasons, consumption exhibits diminishing marginal utility. Meanwhile, the distutility of supplying labor is convex; like climbing a stairs, it gets harder as the level rises. This way, the consumer will want to spread labour supply over time. In this environment, therefore, consumers desire to spread consumption over time, but also wants to spread the labour over time. This eagerness to spread labor supply over time depends on how convex marginal disutility of labour is. For instance, the fact people take long vacations in summer and work five day weeks suggests labor disutility is not *that* convex.

Most importantly, labour is used to purchase consumption. You can think of labour as a means of attaining consumption. That's why people work. The fundamental tradeoff here is between labour, which provides disutility, and consumption which provides utility. For this reason, one key determinant of labour supply is how quickly diminishing marginal utility sets in. To see this tradeoff, suppose the budget constraint is:

$$c = wl + d,$$

where w is the real wage and d is other income, say dividends. For each period, consumer now solves:

$$\max_{c,l} u(c) - v(l) \qquad \text{subject to} \qquad c = wl + d.$$

I can maximize this using the chain rule, noting the dependence of c on w. This gives the first order condition:

$$u'(c)\frac{dc}{dl} - v'(l) = 0 \Rightarrow u'(c)w = v'(l)$$

This is static neoclassical labor/leisure optimality condition, and is generally written as:

$$wu'(c) = v'(l), \tag{1.1}$$

that is, the real return (in terms of utility) to supplying an extra unit of labor is equal to the marginal disutility of labor. The real return to an extra unit of labour is just the wage multiplied by marginal utility. Namely, the real wage indicates how many goods I can purchase by supplying an extra unit of labour. And multiplying this by marginal utility, $u'(c)$, gives the total utility gain. Conveniently, we can think of this as a "marginal gain equals marginal" cost condition.

As always, from the Slutsky equation, we know there are two effects: the income and substitution effects. The income effect says: because you are now richer, you should purchase more leisure (since leisure is a normal good). The more work you are already doing, the greater this effect. In contrast, the substitution effect addresses the question: how should you respond optimally to the change in relative price? A higher wage makes leisure more *expensive*, so people should now bias consumption towards c; i.e., the mainstream consumption goods you buy in stores. Whether the income or substitution effect dominates depends on the form of the utility function. Looking at the optimality condition in (1.1) above, we can see the tension between the two effects. On one hand, the substitution effect is given by the rise in w. All else constant, a rise in w raises $v'(l)$; that is, a rise in w increases marginal disutility. Why? Because labour is rising; this is what the substitution effect dictates. On the other hand, the expression for marginal utility, $u'(c)$ mediates the income effect. All else constant, a rise in the wage causes c to rise. And given that this causes marginal utility to *fall*, the income effect dictates that $v'(l)$ should *fall*; i.e., labour supply should fall.

1.0.1 An Example

To give an example, suppose $u(C) = \frac{C^{1-\theta}}{1-\theta}$, $v(l) = \frac{1}{5}l^2$, and $c = wl$. As we know from studying responses to interest rates, the parameter, θ, mediates how quickly diminishing marginal utility sets in. Following through with this reasoning, θ mediates how quickly people are satiated and how strong income effects are. Now, the consumer solves:

$$\max_{l} \frac{(wl)^{1-\theta}}{1-\theta} - \frac{1}{5}l^2$$

The first order condition is:

$$\frac{w}{(wl)^{\theta}} = l \Rightarrow l^* = w^{\frac{1-\theta}{1+\theta}}$$

Hence, if $\theta > 1$, $\frac{dl}{dw} < 0$, and the income effect dominates. Intuitively, since θ is relatively high, diminishing marginal utility sets in quickly. So why bother working for such low utility gains? What's the point? By contrast, If $\theta < 1$, the substitution effect dominates and labour supply will rise as the wage increases; that is, $\frac{dl}{dw} > 0$. Now, to liven things up, suppose we have a proportional tax rate, t, on labour. The optimality condition is now:

$$\frac{(1-t)w}{((1-t)wl)^{\theta}} = l \Rightarrow l^{*} = ((1-t)w)^{\frac{1-\theta}{1+\theta}}$$

In the case where $\theta > 0$, a high tax rate *raises* labour supply. By contrast, if $\theta < 1$, a high tax rate *reduces* labour supply. Instead of working more as tax rates rise, people now work *less*.[1]

Dynamic Model

In a two period dynamic model, where income is now *endogenous*, this consumer's problem is:

$$\max_{C_1, C_2, l_1, l_1} u(C_1) - v(l_1) + \beta\left(u(C_2) - v(l_2)\right)$$

To keep things simple, assume that the wage rate is constant over time. Denoting savings by S, the income constraint in period 1 is

$$\underbrace{wl_1}_{sources} = \underbrace{S + C_1}_{uses}$$

And in period two is:

$$\underbrace{wl_2 + (1+r)S}_{sources} = \underbrace{C_2}_{uses}$$

Combining these gives the *intertemporal budget constraint*:

$$C_1 + \frac{C_2}{1+r} = wl_1 + \frac{wl_2}{1+r}$$

Then the Lagrangian is:

$$\mathbb{L} = u(C_1) - v(l_1) + \beta\left(u(C_2) - v(l_2)\right) + \lambda\left(wl_1 + \frac{wl_2}{1+r} - C_1 + \frac{C_2}{1+r}\right)$$

[1] An interesting policy question is, what tax rate maximizes the government's tax revenue? To address this question, note that tax revenue in this world is given by $twl^{*} = tw\left((1-t)w\right)^{\frac{1-\theta}{1+\theta}}$.

Differentiating with respect to l_1, l_2, C_1, and C_2, and tidying things up, gives the optimality conditions:

$$u'(C_1) = \beta(1+r)u'(C_2)$$

$$wu'(C_1) = v'(l_1)$$

$$wu'(C_2) = v'(l_2)$$

Taxation

With *proportional* labour taxes of t on labour each period and rates of τ on capital, the intertemporal budget constraint becomes:

$$C_1 + \frac{C_2}{1+(1-\tau)r} = (1-t)wl_1 + \frac{(1-t)wl_2}{1+(1-\tau)r}$$

Hence, the optimality conditions become:

$$u'(C_1) = \beta u'(C_2)(1+(1-\tau)r)$$

$$(1-t)wu'(C_1) = v'(l_1)$$

$$(1-t)wu'(C_2) = v'(l_2)$$

These conditions are effectively instructions dictating the consumer's optimal labour supply. Because the prices faced by the consumer are now different, the optimal labour/leisure choice and savings/consumption choice are both *distorted* and different from before. Of course, the magnitudes of the differences depend on the income and substitution effects.

Chapter 2

The New Keynesian Model

2.1 Introduction

Short-Run Keynesian View

Keynesian view: The interest rate *and* output adjust to ensure goods market equilibrium. Keynesians believe the classical story does not hold in the short-run, though they accept it as a good description of the long-run. Keynesians say that if investment demand falls, this will cause a fall in *production*. In turn, this causes the stock of savings to *fall* and the savings line to shift backwards—so we end up at a high interest rate, even though investment has fallen. Most importantly—and this is key—we will not automatically go the natural rate of interest to clear the goods market when investment falls. Instead, we'll go to some other market interest rate that'll still be "too high."

Today, Keynesians view monetary policy as a means whereby the monetary authority can artificially alter real interest rates (given sticky prices) and thereby affect real economic activity. Old-style Keynesians stressed the role of fiscal policy, but most economists now take a dim view of fiscal policy, mainly as a result of the permanent income hypothesis/Ricardian equivalence arguments. Mainstream macro people and central banks accept take this view as a description of the short-run. After all, if you hear something like "consumer confidence fell, which could lead to a recession,"—and we do all the time!—thats a strongly Keynesian statement, implying output is demand-determined. Anyway, to sum up, the goods market equilibrium condition is:

$$Y = C(r, Y) + I(r),$$

where Y really could be anything. Of course, Keynes key point was that $Y \neq Y_n$ and $r \neq r_n$.

Real Business Cycle Theory

In contrast to Keynesians, real business cycle people say fluctuations in output are *not* due to demand fluctuations. Rather, they are due to fluctuations in potential output. As a result, their benchmark model is the long-run classical model. In recessions, for instance, people *choose* to work less, causing potential to fall. Most controversially, they deny the idea of unemployment—"you can always sell apples at Grand Central," they say. In addition, in the RBC model, money doesn't matter. They regard the empirical relationships between money and output as reflecting reverse causality: economic booms lead to endogenous rises in the money multiplier—due to say balance sheet effects or the FED accomodating increases in money demand—meaning output is causing the money supply to rise. In their support, a lot of the cyclical movements in the money supply are indeed due to changes in the multiplier. Yet Keynesians point to other examples such as the Volcker recession, which was a clear case of the FED reducing the money supply intentionally to *induce* a recession to bring down inflation. In turn, output fell sharply, strongly suggesting money caused output. Taking this as an exogenous "experiment," Keynesians say this event—among others—proves their case.

2.2 The Model

The New Keynesian model is the benchmark model used by central banks and most economists today. Ironically, it was spawned in response to RBC theorists, who claimed—with some justification—that old traditional Keynesian models—like the ISLM or Keynesian cross—were too ad-hoc to be taken seriously and in particular, didn't stem from fundamental microeconomic relationships. The key features of the model are

1. Consumer optimization, where labour and consumption decisions are results of consumer optimization.

2. Firm optimization, which leads to labour demand and optimal pricing/production. For simplicity, there is no capital. There is an imperfectly competitive product market and firms choose prices. However, they take wages as given; these are determined in the labour market, where wages are flexible.

3. Market Clearing/Goods Market Equilibrium. The model is general equilibrium, in that all markets will clear.

4. There is a representative household, N monopolistically competitive firms, and a government. The role of the government is unimportant here.

5. Demand-Determined Output (up to a point anyway): production ultimately responds to demand.

6. Because price-stickiness is central to money nonneutrality, it plays an important role in the model. There is time dependent pricing (not state dependent, which would make prices dependent on the state of the economy.) At any point, some prices will be fixed (regardless of events): firms irregularly set prices. We will rationalize this via "menu costs."

7. To have pricing decisions, we need price setters. Monopolistic competition. (Stark contrast to the price taking assumption under perfect competition, which is more or less standard in modeling the long-run.)

8. The fact $p > mc$ is key. This means there is the potential for demand determined output. Because firms' prices are initially higher than marginal cost,

they will find it optimal to increase production in response to an increase in demand. There is certainly leeway to do this.

9. We need to get back to long-run level of output/potential. The *natural rate hypothesis* holds. The Phillips curve relationship will bring us from the short run to the long run.

The model has three key equations

- The New Keynesian IS Curve which comes from the equilibrium Euler equation plus some exogenous sources of demand (say, exports.)
- The Taylor Rule, which determines the interest rate. Fed sets rates. This does away with LM curve. Money supply endogenous; the FED just adjusts the money supply to hit the rate dictated by the Taylor rule. (You could think of the money market equilibrium condition, $M^s = M^d = L(i, y)$, in the background, where the FED is adjusting M^s to hit its target i.) Importantly, we ignore issues relating to falls in the money multiplier etc.
- The New Keynesian Phillips curve. This will describe the adjustment from the recession/boom back to potential.

2.3 The Household

The household maximizes:

$$E_0 \sum_{t=0}^{t=\infty} \beta^t \left(\frac{C_t^{1-\theta}}{1-\theta} - \frac{L_t^{1+\sigma}}{1+\sigma} \right)$$

The important parameters here are θ, which governs diminishing marginal utility of consumption and σ, which mediates the marginal disutility of labour; as such this parameter mediates the household's incentive to smooth labour supply over time. Think of the σ as the "oh my back hurts" parameter: it mediates how painful further hours of work are. The parameter, $\beta = \frac{1}{1+\rho}$, is the consumer's discount factor.

Related to this is the parameter ρ, which is called the *rate of time preference*. Now, the marginal utility of consumption is

$$u'(C_t) = \frac{1}{C_t^{\theta}}$$

The marginal disutility of labour is

$$L_t^{\sigma}$$

Of course, if $\sigma = 0$, people would be *indifferent* to working, say, 100 hours one day and 50 hours a day for two days.

The flow budget constraint at time t—there is one every period—is

$$\underbrace{W_t L_t + (1+i)B_{t-1} + \Pi_t}_{sources} = \underbrace{P_t C_t + B_t + T}_{uses}.$$

Π_t are profits from the firms at time t. We assume the household owns the firms (after all, *someone* owns them, and the profits have to go somewhere). B_t refers to bonds, which I assume are issued by the government to finance its expenditure; i refers to the *nominal* interest rate. At the start of time the number of bonds, B_0, is given. We could aggregate all the flow budget constraints into a intertemporal budget constraint, like we did in the 2 period model. As it is, though, we can maximize the objective function subject to all the flow budget constraints using the technique of Lagrangian multipliers. We can use the Lagrangian technique to solve this, bearing in mind that there is a constraint for each time period. Fortunately it is relatively easy to solve. To solve it, I'm just going to pick just two random periods t and $t+1$ and invoke the familiar conditions from our two period model. Restricting the analysis to two periods like this is without loss of generality. However, when dealing with infinite time, one also must impose a transversality condition to ensure the consumer doesn't continually permit B to grow indefinitely large or negative. This way, the consumer is not permitted to die in debt; in addition it also ensures the consumer consumes everything in the "last period." Leaving positive assets at the end would obviously not maximize utility if the consumer derives utility from consumption; as such, this also acts as an optimality condition.

The form of the utility function is

$$U(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$$

The parameter θ mediates the degree of diminishing marginal utility and hence the consumer's willingness to shift consumption intertemporally. In particular, the parameter, $\frac{1}{\theta}$ is the *intertemporal elasticity of substitution* and is a measure of the consumer's willingness to shift consumption across periods. Because of this, the parameter will determine the households response to interest rate changes. For example, if $\frac{1}{\theta}$ is high—i.e., θ is low—then the consumer will be eager to save in response to increases in interest rates. From now on, I will assume that this is case and that the substitution effect of price changes dominates the income effect. (If you think about it, this makes sense for a business cycle model: the prices changes are typically temporary, making the attendant income effects weak and substitution effects strong.)

Optimality Conditions

The first order condition for consumption in period t is

$$\underbrace{\frac{1}{P_t}u'(C_t)}_{pain} = \underbrace{\beta\frac{1+i}{P_{t+1}}u'(C_{t+1})}_{gain}$$

Think about it: You give up a euro today. Hence you give up $\frac{1}{P_t}$ *goods*. Because the value of those goods was $u'(C_t)$ each, you therefore lose $\frac{1}{P_t}u'(C_t)$ in utility. Next period, you gain $1+i$ back (initial sum plus interest). With that you can buy $\frac{1+i}{P_{t+1}}$ goods, giving you extra utility of $\beta\frac{1+i}{P_{t+1}}u'(C_{t+1})$. You get $u'(C_t)$ in each unit, and since we value the future less, we discount everything with β.

Because the consumer faces uncertainty about the future (the future price level or future consumption) we should put an expectation operator the right-hand side, giving:

$$\frac{1}{P_t}u'(C_t) = \mathbb{E}_t\beta\frac{1+i}{P_{t+1}}u'(C_{t+1})$$

From now on, however, I'll omit this. Because uncertainty doesn't play a significant role in the analysis, this is fine. Tidying up the above and omitting the expectations operator, we get

$$u'(C_t) = \beta\frac{P_t(1+i)}{P_{t+1}}u'(C_{t+1})$$

Because $\frac{P_{t+1}}{P_t} = 1 + \pi_t$, this gives

$$u'(C_t) = \beta\frac{(1+i)}{1+\pi_t}u'(C_{t+1}).$$

Noting that $\frac{1+i_t}{1+\pi_t} \approx 1 + r_t$,[1]

$$u'(C_t) = \beta(1+r_t)u'(C_{t+1}),$$

where $1 + r_t = \frac{1+i_t}{1+\pi_t}$ is the gross real rate of interest. This governs the *path* of consumption. There is an Euler equation each period. (To get the actual *level* of consumption at a point in time, we'd have to combine the Euler equations with the budget constraints. But the steepness of the consumption (i.e., the rate of consumption growth) is the same for everyone. So if you're a millionaire say, then the steepness of your consumption profile will be the same as a poor person; but of course the levels of consumption in each period *will* be different.) Moving on, the labour/leisure optimality condition is

$$\frac{W_t}{P_t}u'(C_t) = v'(L_t) \Rightarrow \frac{W_t}{P_t}\frac{1}{C_t^{\theta}} = L_t^{\sigma}$$

The *transversality condition* is

[1] To see this formally, take logs of both sides to get $\log 1 + r = \log(1+i) - \log(1+\pi_t)$. Then noting the approximation $\log(1+x) \approx x$ confirms that $r = i - \pi$.

$$\lim_{t\to\infty} \beta^t u'(C_t) B_t = 0$$

Basically, this means you don't want to leave any savings (i.e., bonds) left at the end if you value consumption. Of course, if you don't value consumption "at the end," then $u'(C_t) = 0$, and then it's ok to leave positive savings left over (the TVC allows this).

2.3.1 Demand

Note that, given the functional form for utility, $u'(C) = \frac{1}{C_t^\theta}$. Marginal utility is falling in the level of consumption, and the extent to which it falls depends on that important parameter, θ. Substituting this into the Euler equation above gives

$$C_t^{-\theta} = \beta(1 + r_t) C_{t+1}^{-\theta}$$

Taking logs gives

$$-\theta \log C_t = \log \beta + \log(1 + r_t) - \theta \log(C_{t+1})$$

Letting $c_t = \log C_t$

$$c_t = \frac{\rho - r_t}{\theta} + c_{t+1} \tag{2.1}$$

(If there is uncertainty, I should technically write this as $c_t = \frac{\rho - r_t}{\theta} + E_t c_{t+1}$.) In the background, I assume there is also a government, whose expenditure is another source of demand. Letting government expenditure, $\log G_t = g_t$, total (log) demand, d_t is then

$$d_t = c_t + g_t = \frac{\rho - r_t}{\theta} + c_{t+1} + g_t \tag{2.2}$$

Idea is, a rise in the interest rate induces the consumer to consume less today (we assume the substitution effects dominate). But more generally, I could add in other sources of demand such as investment and exports, balance sheet effects etc.

These would likely depend on the interest rate too and therefore would reinforce the inverse relationship between demand and the real interest rate. For simplicity, I assume government expenditure is independent of the interest rate.

ASIDE: Relationship to Long-Run Interest Rates

The log Euler equation is

$$c_t = \frac{\rho - r_t}{\theta} + c_{t+1}$$

This implies

$$c_{t+1} = \frac{\rho - r_{t+1}}{\theta} + c_{t+2}$$

Then substituting this into the first Euler equation gives

$$c_t = \frac{\rho - r_t}{\theta} + \frac{\rho - r_{t+1}}{\theta} + c_{t+2}$$

and tidying up

$$c_t = 2\frac{\rho}{\theta} - \frac{1}{\theta}(r_t + r_{t+1}) + c_{t+2}$$

$$c_t = 2\frac{\rho}{\theta} - \frac{1}{\theta}(r_t + r_{t+1}) + c_{t+2}$$

More generally, solving forward n times gives

$$c_t = N\frac{\rho}{\theta} - \frac{1}{\theta}(r_t + \cdots + r_{t+N-1}) + c_{t+N}$$

Conveniently, we can use the *expectations hypothesis* to write this in terms of the long-run interest rate. From the expectations hypothesis of the term structure:

$$R^{Nl} = \frac{E_t\left(r_t + \cdots + r_{t+N-1}\right)}{N},$$

where R^{Nl} denotes the interest rate for investing in a N-period long-run bond. Hence,

$$c_t = N\frac{\rho}{\theta} - \frac{N}{\theta}R^{Nl} + c_{t+N}.$$

This makes sense. Consumption today depends on the path of future interest rates and specifically the long-run interest rate. For example, if I expect interest rates to soar in two years time, then that'll tend to reduce my consumption today as I save to exploit this opportunity when it arises. Alternatively, long-run rates will rise today, again inducing a fall in consumption today. In our usual, standard version above in (2.1), such a rise in interest rates in two years is captured by a fall in c_{t+1}, so the standard version still captures this effect, albeit not so explicitly. For this reason, we can still write the Euler equation in its usual form.

Keep in mind that, long-rates are crucially important, and the bank can, via "expectations management," affect them by changing short rates and expectations of *future* short rates. This gives the power of monetary policy as extra "kick." The above provides a rationale for why "expectations management" by the central bank is so important. By committing to keep rates low for a while, they can affect the crucially important long-run rates. From now on, however, I will simply assume that it is r_t, the short-rate in period t that affects economic activity and thus will continue to use (2.1) as the Euler equation. This is still value since r_t of course affects long-run rates.

2.3.2 Goods Market Equilibrium

Letting y_t denote production, the goods market equilibrium is $y_t = d_t$. So for goods market clearing, production equals demand. Most importantly, and in contrast to long-run models, we are not even mentioning potential output here. According to this Keynesian short-run analysis, there's no reason whatsoever for $y_t = d_t = y_n$. The *New Keynesian IS curve*—the goods market equilibrium condition—is

$$y_t = \frac{\rho - r_t}{\theta} + c_{t+1} + g_t$$

Most important thing here is the negative relationship between r and y. A lower interest rate leads to a lower level of output/production in equilibrium (in the background, a lower interest rate stimulates consumption, raises aggregate demand, *and then output*.

Now, when output is at its natural rate, the interest rate equals the natural rate. Hence

$$y_n = \frac{\rho - r_n}{\theta} + c_{t+1} + g_t$$

ASIDE: IS Curve in Terms of Output Gaps

Ignore government expenditure for a moment. Then since $y = c + g$ each period, we can write the IS curve as

$$y_t = \frac{\rho - r_t}{\theta} + y_{t+1}$$

In long-run equilibrium, when output is at potential, we have

$$y_n = \frac{\rho - r_n}{\theta} + y_{nt+1},$$

where y_{nt+1} denotes next period's potential output level. Subtracting y_n from y_t then gives

$$y_t - y_n = \frac{1}{\theta}(-r_t + r_n) + y_{t+1} - y_{nt+1}$$

And setting $x_t = y_t - y_n$, we have

$$x_t = \frac{1}{\theta}(-r_t + r_n) + x_{t+1}$$

With this version, we can see clearly how deviations of the interest rate from its natural rate leads to output gaps. This makes it clear that if r_t is set above r_n, the output gap will be negative; i.e., a recession. When the interest rate equals the natural rate, then the output gap is zero. This is why Taylor Rule tries to aim for natural rate *on average*; it is the level consistent with demand equal to *potential*.

Recall that in the short-run the economy will not automatically go to natural rate itself—*the* central Keynesian idea.[2]

2.4 The Firm

There are N monopolistically competitive firms. I assume N is very large so the atomistic firm takes aggregate demand Y and the price level P as given. All firms face downwardly sloping demand curves—the firms are not price takers. Yet they take the wage as given. In this model, the interaction of labour supply (by the household) and labour demand (by firms) determines the wage in the labour market; we assume the wage is flexible. The households's labour supply condition implicitly determines its labour supply. The level of production at any given point will determine labour demand; we say there is a "derived demand" for labour. As a result, labour demand rises in booms, while it falls in recessions.

For now, I'm just presenting the technical features of the model. I'll get the price stickiness etc in a moment. As well, I'm dropping the time subscripts, but I should subscript everything with a t below.

The firms faces demand

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} \frac{Y}{N}$$

So in a boom Y_i will rise, since aggregate demand, Y, is higher (this could be due to a rise in government expenditure or a fall in interest rates by the FED, say). To be consistent with the analysis before, the Y I put here should really be the d_t in Eq. 2.2 above. But forget about the distinction between logs of variables and their

[2]For instance, in the long-run classical model, if consumption fell, then the rise in savings would cause the natural interest rate to fall, which would cause investment to rise. And it would rise to clear the goods market, so we'd end up back at potential again with lower consumption, but higher investment. By contrast, Keynes argued that if consumption fell, *output* (and to a much less extent, the interest rate) would adjust and so, instead, the economy would enter a recession. This fall in aggregate output would cause the *stock* of savings to fall, so interest rates might even be higher after all the adjustments.

actual levels—this distinction has no bearing on the central ideas.

Now, by choosing P_i, the firm implicitly chooses Y_i too. Note that P is the price *level* in the economy, which you can think of as simply the average price set by all firms. Formally, it would correspond to a price index such as the CPI.

What determines demand for the firms product is its *relative price*, not its absolute price P_i. This is what the firm will keep in mind and what we will ultimately try to solve for. Moreover, by setting a relative price, it ensures it's maximizing real profits—what the firm actually cares about. For example, setting a price of 100 in an environment were all other prices are around 1000000 will obviously not maximize *real profits* (i.e., the purchasing power of profits). Notice that, even if firm has higher price, there is still positive demand for their product. We often explain this by saying the consumer has a "love of variety" and would like to purchase a little of everything. Think about it; when you buy yoghurts you might purchase one of each flavor even if some are more expensive than others. At a micro level, this "love of variety" can be rationalized by diminishing marginal utility to individual *goods*; therefore, you'd rather have a strawberry and apple yoghurt rather than two apple ones. Right? The elasticity of demand, η, depends on the substitutability between goods. As we'll see in a moment, this will determine the markup.

The firm's production function is

$$f(L) = L$$

where L is number of workers hired by firm. Note that the marginal product of labour is $MPL = 1$. (To make things more realistic, we could also have $f(L) = AL$, where A denotes productivity.) The production function will determine the marginal cost in this model. Because $A = 1$ and labour is the only factor of production in our basic model, real wages will ultimately determine marginal cost. It follows that if a firm wants to produce Y_i units, it needs to hire Y_i workers. As I said, there is a *derived demand* for labour: it derives fundamentally from the level of aggregate demand.

Now, if a firm charges P_i, the demand for its goods is $\left(\frac{P_i}{P}\right)^{-\eta} \frac{Y}{N}$. The firm's revenue is then $P_i\left(\frac{P_i}{P}\right)^{-\eta}\frac{Y}{N}$. Its labour demand will be then Y_i, and its costs are WY_i. Overall, by setting a price of P_i, the firm's profit is then

$$\Pi_i = P_iY_i - WY_i = Y_i(P_i - W)$$

Substituting in the expression for demand gives

$$\Pi_i = \left(\frac{P_i}{P}\right)^{-\eta}\frac{Y}{N}(P_i - W)$$

The only choice variable is P_i. As noted above, the wage is exogenous to firm, and will depend on developments in the national labour market (in particular, firms' labour demands interacting with household's labour supply).

Maximizing with respect to P_i is[3]

$$P_i = \frac{\eta}{\eta - 1}W$$

In this world, marginal cost equals the wage, so more generally, $P_i = \frac{\eta}{\eta-1}MC$. (It's typical to write this result in terms of marginal cost, so, later on, if I replace the formula above with $P_i = \frac{\eta}{\eta-1}MC$, don't be alarmed.) This is what the firm would charge in a *flexible price equilibrium.* The constant, $\frac{\eta}{\eta-1}$ is the firm's target markup. No matter what, the firm will always seek this markup. At any point in time, this is the ideal price, *no matter what the current markup or level of demand is.* This is a key point to consider, especially when we deviate from equilibrium. Note too that the markup is pinned down by fundamentals in the economy; i.e., the elasticity of demand.

A few points to note here. Most significantly, the firm charges a price that exceeds the marginal cost of production. This is a consequence of the firm's monopoly power. You see, to maximize profits, it considers the "menu of options" given by the demand curve; for example, it could charge a high price and have low demand/output; or a

[3] Regarding second order conditions, $\Pi_i'' < 0$ (i.e., the second order derivative with respect to P_i), so this is indeed a maximum.

low price and high demand/output. Ultimately—as the maths tells us—what maximizes its profits is setting a price of $P_i = \frac{\eta}{\eta-1}W$ and producing $Y_i = \left(\frac{\frac{\eta}{\eta-1}W}{P}\right)^{-\eta}\frac{Y}{N}$. (To get the latter, just substitute the firm's optimal price into its demand curve.) To give a concrete example, suppose the maths tells us that a firms optimal price is 10, and its optimal quantity is 30. Suppose its marginal cost at that point is 8. Of course, the maths tells us that producing more will *reduce profits*. Why? Well, if the firm wanted to produce 32, say, it'd have to reduce its price to 9, say. So, it gains an extra 9, but *loses* 1 on all existing units. In this case, therefore, its marginal revenue from producing an extra unit would be $9 - 30(1) = -21$. Hardly an attractive option, is it?

Because $P > MC$ in monopolistic competition, output is below the socially optimal level. An implication of this is that, *all else constant*, people would be happy to pay P' where $MC < P' < P$, say, and both the firm and these people would be made better off. Yet this transaction never occurs. Technically, the monopolistically competitive equilibrium is not Pareto optimal, and the First Welfare Theorem does not hold. By contrast, in the ideal market structure of perfect competition, the firm always produces where $P = MC$.

Moving on, the firm is mainly concerned with is relative price, since that is what determines its demand. To get the firm's desired relative price, just divide the formula above by the price level P:

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} = \frac{\eta}{\eta - 1}\frac{MC}{P}$$

This is firms *ideal* relative price if it were free to adjust. So if the real wage, $\frac{W}{P}$, rose, the firms desire price, P_i, would also rise.

2.5 Equilibrium

From above we have

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P}$$

Now, in equilibrium, we assume all firms are the same, and face the same marginal costs. In a symmetric equilibrium, therefore, all firms will charge the same price, so $P_i = P$. So, in equilibrium, each firm has a relative price of one. Moreover, each firm's price equals the price level. If all students in class set a price of 10, then the price level in the class will obviously be 10. Thus in a symmetric equilibrium,

$$\frac{P_i}{P} = 1 = \frac{\eta}{\eta - 1} \frac{W}{P}$$

This implies the equilibrium real wage is

$$\frac{W}{P} = \frac{\eta - 1}{\eta}$$

Note in particular that this is less than 1, and 1 is the marginal product of labour. The fact that the firm pays the worker less than what he produces—i.e., $\frac{\eta-1}{\eta} < 1 = MPL$—is of course the source of his profits (and is another way of saying price exceeds marginal cost). The firm makes a profit of $1 - \frac{\eta-1}{\eta}$ per unit. For instance, if $\eta = 3$, then the real wage will be $\frac{2}{3}$; so the firm will earn $\frac{1}{3}$ on each unit produced.

Now, equilibrium production by any firm is then $\left(\frac{\frac{\eta}{\eta-1}W}{P}\right)^{-\eta} \frac{Y}{N}$ So with equilibrium real wage of $\frac{W}{P} = \frac{\eta-1}{\eta}$, equilibrium production by any firm is $\frac{Y}{N}$. Hence, by symmetry, total equilibrium production in the economy will by Y. This will be our Y_n. Note that this is also equal to equilibrium aggregate demand. Yet, as things stand, this could be *anything*.

So what is equilibrium output, anyway? Well, production was entirely determined by labour; this was the only factor of production. There's simply no other way to get more output, given the production function. So, fundamentally, we must ask how much is the household willing to supply at a real wage, $\frac{W}{P} = \frac{\eta-1}{\eta}$. But how do we find out what the worker supplies at a given wage? Well we go to the labour supply condition i.e., the first order condition for labour. So just find out what labour supply is at equilibrium wage, $\frac{\eta-1}{\eta}$. We can get this from the labour supply curve. I wont go into details here, but just think of picking the point on the labour supply curve where real wage is $\frac{\eta-1}{\eta}$.[4] This will pin down equilibrium labour supply and hence equilibrium output/production. This level will be the level of *potential or natural* output. This is determined by fundamentals and the interaction of household/firm maximization. Because of monopolistic competition and the fact $P > MC$, though, output will be below its socially efficient level.

That's it. We now have equilibrium real wage, equilibrium output/production/demand and equilibrium labour supply. All actions are consistent. Labour demand equals labour supply. Aggregate demand equals the level of production. We havent look at the breakdown of output between consumption and government expenditure, but we know that the goods market will clear: $Y = C + G$. To get the natural rate of interest, we go back to the Euler equation $u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$. In equilibrium, we know that $C = Y - G$,where Y is the natural rate of output and G is some constant. Assume too that the natural rate of output is constant, which must be true here, since there's no obvious way for it to rise over time.[5] Substitution $C = Y - G$ into the Euler equation gives

$$u'(Y - G) = (1 + r)\beta u'(Y - G)$$

and so $r_n = \frac{1}{\beta} - 1$.

If we introduced money, the usual money market equilibrium condition must be satisfied in equilibrium[6]

[4] We know therefore that, in equilibrium, $\frac{\eta-1}{\eta}\frac{1}{C^\theta} = L^\sigma$. But labour supply must equal output, so $\frac{\eta-1}{\eta}\frac{1}{C^\theta} = Y^\sigma$. Now, if we assume that consumption C is some fraction γ of output, Y, in equilibrium (so government expenditure makes up a fraction $1 - \gamma$), then solving $\frac{\eta-1}{\eta}\frac{1}{(\gamma Y)^\theta} = Y^\sigma$, will give us equilibrium output/labour supply. See?

[5] If the production function was $Y = AL$ and technology, A, was growing, then the natural rate of output would rise over time.

[6] Of course, to generate a demand for money like this, we'd have to put money in the utility function, but rest assured, we can easily do that and derive a money demand from microfoundations.

$$\frac{M}{P} = L(r_n, Y_n)$$

Here Y_n and r_n have already been determined. In this model, if prices are flexible, a rise in M just rises P, and *money is still neutral.* Notice that P must adjust since r_n and Y_n have already been nailed down by fundamentals.

2.6 Increase in Demand at Potential

We now turn to see how output can be demand determined in the short run. Central to this story is price rigidity. For now, I simply assume prices are fixed. With prices fixed, we will see how "money matters." Note, however, that the increase in demand, could be attributable to anything—such as a rise in government expenditure, consumers feeling wealthier etc.[7] For now, Ill focus on the case where the money supply increases, which causes a fall in the real interest rate and this causes an increase in aggregate demand to $Y > Y_n$. To get the intuition, it might be convenient to simply thing of a "helicopter drop" of money, which raises demand to Y too. Anyway, demand is now

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} \frac{Y}{N}$$

Each firm now faces increase in demand. Its prices are fixed. But it can increase profits by producing more to meet demand, since $P_i > MC$. *Given* prices are fixed, this is its profit maximizing strategy. In other words, subject to the fact that prices are fixed, this is the best the firm can do. For this to work, of course, marginal costs must not rise above the firm's fixed price. As a result, labour demand increases, and since all firms are symmetric, labour demand increases nationally. From the production function, we know that this is the only way the firm can produce more.

[7] Currently, for instance, demand has likely fallen due to—among other things—consumers perceptions of future income being lower.

To increase labour supply, real wages must now rise. Workers dont like work, and they're already quite happy at potential. Given C_t this labour/leisure optimality condition implicitly defines a labour supply function:

$$\frac{W_t}{P_t}\frac{1}{C_t^\theta} = N_t^\sigma$$

Or more explicitly,

$$L_t = \left(\frac{W_t}{P_t}\frac{1}{C_t^\theta}\right)^{\frac{1}{\sigma}}$$

By the permanent income hypothesis (i.e., the Euler equation), any temporary increase in the wage should be smoothed over the entire lifetime. So the effect on consumption today of a temporary increase in the wage will be small. After all, households know this is merely a *transitory* increase as a result of the business cycle. Therefore, it's ok to treat the above as an increasing relationship between the real wage and labour supply (assuming the real wage increase is transitory and therefore doesnt affect C too much.) Point is, to increase L, $\frac{W_t}{P_t}$ must rise. By how much? This depends of course on σ.

Clearly this is not good for the firm. Ideally, now, they'd like to raise price, which you'll recall is increasing in marginal cost—which, alas, has just shot up. Specifically:

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{W}{P}$$

But more generally, for *any* arbitrary price and marginal cost:

$$\frac{P_i}{P} = \text{MARKUP}\frac{W}{P}$$

But because P_i is fixed, and $\frac{W}{P}$ has <u>risen</u> (above $\frac{\eta-1}{\eta}$), the markup now deviates from desired one, $\frac{\eta}{\eta-1}$. In particular, markup is now less than equilibrium markup $\frac{\eta}{\eta-1}$. This is not good for the firm! Remember, they always desire that target markup $\frac{\eta}{\eta-1}$

Because the price if fixed and the real wage has risen (above $\frac{\eta-1}{\eta}$, the target markup falls. Given firm's profit maximization objective, this suboptimal markup is unsustainable. If they desired this lousy markup, they'd have charged that at equilibrium. But they didn't. And as they say, *things that cant go on forever don't.* In addition, we must assume the real wage doesn't jump up too much. The firm will only be happy to increase output as long as $P > MC$. If the marginal cost jumps up to $MC < P$, then they will not meet all of the increase in demand. Each firm's target price has risen. Eventually (in the proverbial "long-run"), all firms start raising prices, causing the *price level*, P, to rise. In turn, this causes real demand to fall. From the money market equilibrium condition, $\frac{M}{P} = L(r, y)$, the rise in the price level, will cause the supply of real money balances, $\frac{M}{P}$, to fall. This causes the real interest rate to rise and output to fall. (Somewhat more intuitively, you could say that if some firms raise their prices, then that'll leave their customers will less money to spend on *other* firms, meaning the level of real aggregate demand will fall.) As a result, labour demand falls and real wages rise. This process will continue until we get back to potential, and ultimately, the price level will rise in proportion to the rise in the money supply. But money will be nonneutral until they do so.

Comments

Rather that saying "money is nonneutral" in the short run, we know have a story, a mechanism, through which this happens. It shows how the FED can temporarily affect output. Moreover, we also have the "money is neutral in the long run" story too. We've seen how the economy adjusts back to the potential equilibrium; this is called the *natural rate hypothesis.* In particular, we've seen how starting at

$$\frac{M}{P} = L(r_n, Y_n)$$

an increase in M, with P fixed causes Y to rise above Y_n.

Exactly why this happens should be clear. Sticky prices are central to the story; they are the propagation mechanism of the model. This is consistent with FED affect rates temporarily and the reality that FED cant affect activity forever. As long as prices are sticky, the FED can stimulate economy. Thus booms and busts can be quite persistent, especially if prices adjust sluggishly.

Note that when demand increases, production increases too to meet that level. Most importantly, we have rationalized the idea of *demand-determined* output. We've seen how changes in demand induce proportional changes in output. And just to be clear, the increase in demand could have been generated by changes in government expenditure or changes in consumption (or, in a more general setting, investment, exports, and so on.) Of course, to get this, we assume marginal cost doesn't rise above the preset price.[8]

Imagine now if the money supply increases and prices are flexible. In a *flexible price* economy, we assume the above adjustment process happens in an instant. The increase in labour demand instantly pushes up real wages, and firms instantly increase their prices and demand falls back to potential.

Suppose the economy was at potential and there was a sudden rise in demand—due to say, a fall in consumption expenditure as a result of consumers' expectations of higher future income. The story would be the same as outlined above. Therefore, this analysis provides a rationale for demand-determined output regardless of its source.

Recessions

Content yourself that the opposite situation occurs when the money supply falls (or, for that matter, if government or consumption expenditure fell). If say government expenditure suddenly fell, then demand would fall and firms, keeping prices fixed, would simply cut back production. In turn, labour demand would fall, along with real wages. In this case, then, the firms keep their prices *too high.* As above, eventually, they'll change their prices to a lower level, which will increase aggregate demand again. Of course, this kind of story provides a rationale for the FED to

[8]Note that we must use imperfect competition to model this. If $P = MC$ and marginal cost rises, then the firm will *not* meet the demand; otherwise it would suffer losses. For this reason, to model money nonneutrality, we need some form of imperfect competition.

lower rates in recession. Point is, the lower interest rates would raise consumption demand and therefore raise aggregate demand (and hence production) again. Of course, Keynes recommendation was for the government to increase expenditure to make up the short-fall in demand; that is, the government, via fiscal policy, should increase expenditure in recessions.

A Note on the Labour Market

The labour market is a little strange in the model, but fortunately that has no bearing on anything important. The way I've modelled it, we just have a single household supplying labour hours. Implicitly, I've assumed the household spreads those working hours and bodies over all firms. Because of this, there is no unemployment in the model; the household simply increases and decreases labour market activity as the firm "seduces" it into doing so by changing wages. that is, its just the same people reducing and lowering hours worked all the time. There are no new people entering the workforce. You might suspect—and you'd be right—that labour markets are rather different in reality. However, none of this really matters for the what the model seeks to say.

2.6.1 Real and Nominal Rigidity

A crucial question for the model is, why are prices so sticky anyway? If prices are flexible, the entire model falls apart. There are two ways to rationalize sticky prices. The first is *nominal rigidity.* This refers literally to some "menu costs" of changing prices. Realistically, however, this would have to be fairly large. If marginal costs rise sharply in booms, then the incentive for firms to raise prices is surely large and might well overwhelm any "menu cost." Therefore, although we certainly need some nominal rigidity, we really need what's called *real rigidity* too.

What is real rigidity? Real rigidity measures the degree to which marginal costs change due to fluctuations in output. Real rigidity refers to little change in firms marginal costs—the source of price changes—over the business cycle. Real rigidity refers to ways of of rationalizing little incentives by firms to change prices (and hence price rigidity even in the face of a small "menu cost.") In turn, a greater degree of real rigidity leads to greater money nonneutrality.

To rationalize sticky prices, we need some nominal *and* real rigidity. The essential idea behind real rigidity is this: is there something that prevents marginal costs from rising a lot in booms (or conversely, prevents marginal costs from falling a lot in recessions)? Throughout, and without loss of generality, I will stick to the boom case. Before going on, recall the formula for the firms optimal price at any point

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{MC}{P}$$

where $\frac{MC}{P}$ denotes the firms real marginal cost. Note further, that when we have the production function, $Y = AL$, the marginal cost is $\frac{W}{MPL} = \frac{W}{A}$. Well, here are some sources of real rigidity, but this list is by no means exhaustive

1. Increasing Returns to scale or some kind of "learning by doing." Basically, in a boom productivity rises. As a result, the firm's marginal cost falls. So if we have the real wage rising, but productivity rising simultaneously, then real marginal costs wont rise as much (and could in fact fall!) Certainly, this reduces the firms' incentives to change their prices. (By contrast, in a recession, this will cause productivity to *fall*. In turn, this reduces the firm's incentives to *lower* its price.)

2. Implicit Contracts: Say firms make a deal with workers to keep their real wages stable over time. Then, their wages wont rise in booms and wont fall in recessions. If workers say have little access to capital markets, then this will act as an insurance for them, and help them smooth their consumption over time. On the other hand, this would make firms profits more variable. This kind of real rigidity could be rationalized by assuming firms are less risk averse—or indeed risk neutral—than workers.

3. Balance Sheet effects. If property and equity prices rise in booms, then this will mean firms can offer banks more collateral for loans. Because the value of

their collateral has gone up, this means a) banks will be more willing to lend and b) banks will charge the firms a lower risk premium in lending to them (formally, the firm's "external finance premium" falls in booms). By the way, this tendency for loans to rise in good times and the cost of capital to fall is called the "financial accelerator"; namely it tends to make booms even more expansionary and recessions more contractionary. Overall, this lowers firm's real marginal cost, attenuating the their desire to raise prices.

4. Cyclical Variation in Markups: If the firm's elasticity of demand rises in booms due to, say, greater competition, then their desired markup will fall. The interaction of a rising real wage and a lower desired markup would, on net, lead to less upward pressure on prices.

5. Outward shifts in the labour supply curve. If, for example, labour supply increases in a boom for any given level of the wage, that that will mitigate any increase in the equilibrium real wage. Say if one spouse if working long hours in a boom; then, bored at home, the other spouse heads out to work too.

Note that if there were, say, trade unions who increased wage demand in a boom (due to more bargaining power as a result of "tighter" labour markets), then there would be more upward pressure on real wages. Of course, this would *lower* the degree of real rigidity, making it harder to rationalize sticky prices. With such a feature, prices would surely be more flexible, attenuating the power of monetary policy.

Note finally why we need some nominal rigidity too. If we didnt have nominal rigidity, prices would still change—unless the degree of real rigidity miraculously caused marginal costs to stay *constant.* Realistically, the real rigidity will cause marginal costs not to rise *as much as they "should"*, and then the nominal rigidity or menu cost makes it optimal for the firm to leave prices as they are.

2.7 The New Keynesian Phillips Curve

Now, we want to formalize price setting. In the basic New Keynesian model, I've assumed all prices are fixed in the short-run. Yet, in reality, some firms are always adjusting, while others are not. This does not affect the fact that money is nonneutral, since the *price level* is still sticky. But it *does* mean that the price level will adjust somewhat to today's output gap. The New Keynesian Phillips curve gives a description of how the price level and hence inflation changes over time. To give the intuition, Ill give a baby example first.

From above, the optimal pricing rule for firm i is

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{W}{P}$$

For now, just assume the marginal cost comes completely from real wages, as it did above in the model.

Now, take logs to get

$$\log P_i - \log P = \log \frac{\eta}{\eta - 1} + \log \frac{W}{P}$$

As is standard, write lower case letters for logs; e.g., $\log X = x$ etc:

$$p_{it} = p_t + \log \frac{\eta}{\eta - 1} + \log \frac{W}{P}$$

To give a baby example, suppose firm i sets prices today for two periods, so they are "locking" themselves in now. (In the background, imagine there is some "menu cost" to changing prices, making it optimal for the firm to keep prices fixed for 2 years.) Now, the optimal price in period 1 for firm i is:

$$p_{i1} = p_1 + \log \frac{\eta}{1 - \eta} + \log \left(\frac{W}{P}\right)_1$$

Firm i's expected optimal price next year is

$$E_1 p_{i2} = E_1 p_2 + \log \frac{\eta}{1 - \eta} + E_1 \log \left(\frac{W}{P}\right)_2$$

where E_1 denotes expectation as of time 1. An obvious solution to the firm's problem is to set the price today equal to an average. That is, the forward-looking firm—who has rational expectations—sets a price today of

$$p^* = \frac{p_{i1} + E_1 p_{i2}}{2}$$

This way, we come as close as possible to maximizing profits each periods. Note, however, prices are now suboptimal each period.

Substituting:

$$p^* = \frac{1}{2}\left(p_1 + \log\frac{\eta}{1-\eta} + \log\left(\frac{W}{P}\right)_1\right.$$

$$\left. + E_1 p_2 + \log\frac{\eta}{1-\eta} + E_1 \log\left(\frac{W}{P}\right)_2 \right)$$

today. More generally, we could extend this to arbitrarily many periods. (If the firm cared less about future profits, we'd have a weighted average, with more weight on the present; but forget about this for now (more on that later.)) There is a lot of insight here already. First, the price set today depends on expected future real marginal costs. For example, if the firm expects high real wages next period, that will raise *today's* price (and hence price *level*). Second, the price set today will depend on the expected *future* price level, $E_1 p_2$. Thus, if for some reason, the firm expects higher prices on average in the economy next period, that will raise the firm's optimal price *today.* For example, if the firm predicts a large depression next period, the firm might predict other firms—who might be free to adjust prices—will lower their prices next period, thereby causing a fall in the *price level.* A dramatic fall in money supply is one reason this might happen. Hence expectations of future monetary policy affects price-setting behaviour *today.*

2.8 Derivation of New Keynesian Phillips Curve

From now on, its useful to think of a more general production function where MC could incorporate a range of inputs such as oil or the cost of capital.

To start with, from above

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{MC}{P}$$

Taking logs and imposing equilibrium condition, $P_i = P$:

$$0 = \log\frac{\eta}{\eta - 1} + \log\frac{MC}{P}$$

Using log rules

$$\log\frac{MC}{P} = \log\frac{\eta - 1}{\eta}$$

Denoting the equilibrium log real marginal cost by $\widetilde{\log \frac{MC}{P}}$, we have $\log\frac{\eta-1}{\eta} = \widetilde{\log \frac{MC}{P}}$

More generally, we have

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{MC}{P}$$

$$\log P_i - \log P = \log\frac{\eta}{\eta - 1} + \log\frac{MC}{P}$$

Using log rules again:

$$p_{it} = p_t - \log\frac{\eta - 1}{\eta} + \log\frac{MC}{P}$$

But from above we know that $\log\frac{\eta-1}{\eta} = \widetilde{\log \frac{MC}{P}}$. Substituting then gives

$$p_{it} = p_t + \log\tfrac{MC}{P} - \widetilde{\log\tfrac{MC}{P}}$$

$$p_{it} = p_t + \log\tfrac{MC}{P} - \widetilde{\log\tfrac{MC}{P}}$$

This makes sense; your target price deviates from equilibrium if real marginal cost deviates from equilibrium.

Realistically, and as should be clear from the basic model above, $\log \frac{MC}{P} - \widetilde{\log \frac{MC}{P}}$ is proportional to output gap, $y_t - y_n$. That is for some $\alpha > 0$:

$$\log \tfrac{MC}{P} - \widetilde{\log \tfrac{MC}{P}} = \alpha(y_t - y_n)$$

In other words, when output goes above potential, real marginal cost rises above its equilibrium level (i.e., the level at potential). It follows, therefore, that the firms optimal (log) price at any time is:

$$p_{it} = p_t + \alpha(y_t - y_n)$$

If the firm were free to change prices, *this* is what it would change to. Several points are worth noting here. The parameter, α mediates the degree to which marginal cost responds to output. Because changes in marginal cost are the underlying sources of price level changes (and hence inflation) in the model, deviations in output from potential play a central role in pricing pressure. In this sense, α partly mediates the degree of *real rigidity* in the economy: how responsive is price to deviations of output from potential? For example, if α is low, then there is a lot of real rigidity; prices don't change much as output varies over the business cycle. Of course, a low α could also reflect a high degree of nominal rigidity too. Keep in mind, as well, that the extent to which wages—and hence marginal costs—rises depends on such factors as the elasticity of labor supply. Point is, the parameter α relates to fundamentals in the economy.

Unemployment and the NAIRU

Denoting the NAIRU by u_n we also know that

$$\boxed{y > y_n \Leftrightarrow u < u_n \quad \text{and} \quad y < y_n \Leftrightarrow u > u_n} \tag{2.3}$$

So we could alternatively write (for some $\beta < 0$)

$$\log \tfrac{MC}{P} - \widetilde{\log \tfrac{MC}{P}} = \beta(u_t - u_n)$$

Keep the above in mind. As we shall see, it will permit us to write the New Keynesian Phillips curve in terms of unemployment rather than output.

The NAIRU is basically the long run average rate of unemployment. It's about 5% in the U.S. today, almost half that of continental Europe. It's important to note that the NAIRU are NOT attributable to business cycle fluctuations. Indeed, the NAIRU is the unemployment rate *at* potential i.e., when there's no fluctuations. It's due to factors endemic to the economy such as government policies and labor market institutions—-the strength of trade unions, minimum wage legislation, the generosity of welfare payments, how hard it is to hire and fire, and so on. All of these either reduce the incentive for job search or lead to non market-clearing wages—and hence high equilibrium levels of unemployment. The rigidity of labour markets in continental Europe is often referred to as *Eurosclerosis*. Finally any increase in unemployment above the NAIRU is said to be *cyclical* or induced by recessions. Yet this is a short-run issue. People have lost jobs due to some recession; eventually equilibrium will be restored and the economy will revert back to the NAIRU

2.9 New Keynesian Phillips Curve

We assume now that *some* firms change prices each period. Opportunities to change prices are time-dependent.[9] In particular, each firm faces some given and fixed probability of changing price each period. For example, if that probability is .1, then there will, on average, be little opportunity to change prices. That is, they'll change prices infrequently. On average, the firm's price in this case will be fixed for ten periods. Crucially, the forward looking firms—with rational expectations—take account of this when the have an opportunity to change. If this is a .1 world, say, they'll set prices keeping in mind developments far into the future. Reason being, when they change their prices they know they're "locking themselves in" for a long time. Therefore, when they're setting prices today, they'll put a good deal of weight on the future optimal prices, since they know they mightn't get another chance to

[9] With *state-dependent* pricing, firms change prices depending on the state of the economy.

reset for a while. So they'd better get in right and pay a lot of attention to the future when setting prices *today.*

Overall, when firms get an opportunity to change prices, they'll consider a) optimal prices in future b) probability of changing again. If there's a greater chance of an opportunity to change prices in the future, they'll place less weight on future prices when they setting prices today. Best off to wait till the future, when they're likely to get another chance to change. But if opportunities to change are rate, then they'll naturally place more weight on the future when they're setting prices today.

As shown above, the firm's optimal (log) price at any time t is (assuming potential is constant)

$$p_{it}^* = p_t + \alpha(y_t - y_n)$$

Letting $x_t = y_t - y_n$, we have

$$p_{it}^* = p_t + \alpha x_t$$

A fraction δ set their price in current period. This is also the probability of getting a chance to change each period; it measures the degree of *nominal* rigidity (recall that *real* rigidity measures the *extent* to which firms want to change their prices.) So a fraction $1 - \delta$ leave things be; they don't get a chance to change. Note that the price level in a given period is the average of all prices in the economy; here it is a weighted average of those who change and those who keep their stale prices from last period. So (log of) price level is

$$p_t = \delta p_t^* + (1 - \delta)p_{t-1}$$

Note now that changes in logs are equal to time derivatives. By the chain rule $\log P_t - \log P_{t-1} \approx \frac{d \log P_t}{dt} = \frac{d \log P_t}{dP_t} \frac{dP_t}{dt} = \frac{\frac{dP_t}{dt}}{P_t}$. Then, taking $\log P_{t-1} = p_{t-1}$ from each side of the above gives

$$\pi_t = \delta \pi_t^* \tag{2.4}$$

We'll come back to this in a moment. Recall the optimal price formula from above: any firm changing prices in period t will charge

$$p_t^* = p_t + \alpha x_t$$

Take p_{t-1} from each side to get

$$\pi_t^* = \pi_t + \alpha x_t$$

By choosing prices, firms are implicitly choosing inflation rates. Thats all this means. So optimal prices implicitly give optimal inflation rates. Rather than thinking of firms setting prices, in the derivation of the NKPC, we think of firms setting inflation rates.

Now consider a firm changing price today and locking itself in. What does it do? Analogous to the baby example above, the firms sets the current prices as a weighted average of all the future optimal prices. The weights depend on two factors. First, how much does the firm care about future profits? This will depend on the firm's discount factor, $\phi < 1$. Second, what are the chances the firm will get to reset prices in the future? Obviously, if the firm has lots of chances to reset (i.e., δ is high), then it'll place less weight on expected optimal future prices (since it correctly assumes it'll surely get an opportunity to change soon). Thats why the $(1-\delta)$s appears in front of them. Thus, for example, if opportunities to change come frequently (i.e., δ is high), the firm will downweight future prices highly by placing a $1-\delta$ in front of future optimal prices. Accounting for all these factors, when the firm gets an opportunity to change, it sets a price of

$$\pi_t^* = \pi_t + \alpha x_t + (1-\delta)\phi(E_t(\pi_{t+1} + \alpha x_{t+1}) +$$

$$+(1-\delta)^2\phi^2(E_t(\pi_{t+2} + \alpha x_{t+2}) + (1-\delta)^3\phi^3(E_t(\pi_{t+3} + \alpha x_{t+3}) + ...,$$

where this summation is infinitely long. Note that if $\delta = 1$, then firms will only take account of today's output gap when setting prices today. This makes sense;

itll have another chance to change next period and will be able to respond best to next periods developments. The ϕs just capture the fact that the firm cares less about future profits as ϕ falls; hence itll pay less attention to "getting things right" in the future when its changing prices today. Instead, it cares more about "getting it right" *today* and thereby maximizing today's profits.

Now from (2.4) above we know that:

$$\pi_t = \delta \pi_t^*$$

Substituting our expression for the optimal price into the above gives

$$\pi_t = \delta \left(\pi_t + \alpha x_t + (1-\delta)\phi(E_t(\pi_{t+1} + \alpha x_{t+1}) + (1-\delta)^2 \phi^2 E_t(\pi_{t+2} + \alpha x_{t+2}) + ... \right) \tag{2.5}$$

And from this, get $E_t \pi_{t+1}$ and hence $(1-\delta)\phi E_t \pi_{t+1}$:

$$(1-\delta)\phi E_t \pi_{t+1} = (1-\delta)\phi\delta \Big(E_t(\pi_{t+1} + \alpha x_{t+1}) + (1-\delta)\phi E_{t+1}(\pi_{t+2} + \alpha x_{t+2}) + ... \Big) \tag{2.6}$$

Now, subtract the above from (2.5) to get

$$\pi_t - (1-\delta)\phi E_t \pi_{t+1} = \delta \pi_t + \delta \alpha x_t$$

Tidying this up

$$(1-\delta)\pi_t = (1-\delta)\phi E_t \pi_{t+1} + \delta \alpha x_t$$

$$\Rightarrow \pi_t = \frac{\alpha\delta}{1-\delta} x_t + \phi E_t \pi_{t+1}$$

Then writing the output gap more formally:

$$\pi_t = \frac{\alpha\delta}{1-\delta}(y_t - y_n) + \phi E_t \pi_{t+1}$$

We're done. The intuition? Really, this is based on reasoning in baby example above. First, todays inflation depends on today's output gap, $y_t - y_n$. Recall how the output gap was a proxy for the deviation of *real* marginal cost from its equilibrium value. So a high output gap is always coincident with high marginal cost. Namely, a large output gap today will lead to upward pressure since marginal costs are rising (recall money supply story above.) Notice how α appears; as we know, a high α makes marginal cost more sensitive to output gaps and will lead to greater upward pressure on prices for any *given* output gap. Second, today's inflation will depend on today's expectation of future inflation. As noted above, those setting prices today consider what happens to the price level in future period. For example, if they expect a high price level next period, to maintain real profits—as they "lock" themselves in—they will set a higher price today. Equivalently, if they expect high inflation next period, they will set high prices *this* period, leading to high inflation *today.* Note that output gaps often come from money growth, so money growth is usually lurking in the background of the NKPC.

Note that as δ rises, and so there are more firms changing each period, the coefficient on the output gap rises. This makes sense: as the proportion of firms changing today increases, price increases will respond more to today's output gap. If firms are changing prices more often, then there will be more inflation pressure for any given output gap. Realistically, too, δ surely depends on the rate of inflation (by the Lucas Critique).

Finally, don't confuse *disinflation* and *deflation.* Deflation is when prices actually *fall*; a sufficiently large output gap will lead to deflation. Disinflation, by contrast, is a reduction in the rate of inflation, say from 15% to 10%. So, under *disinflation*, we can typically have prices still *rising.* Think of it like this: deflation is walking backwards, disinflation is slowing down, and hyperinflation is sprinting.

2.10 Three Equation Model

$$y_t = \frac{\rho - r_t}{\theta} + c_{t+1} + g_t$$

$$r_t = r_n + \gamma(y_t - y_n) + \beta(\pi - \overline{\pi}).$$

$$\pi_t = \frac{\alpha\delta}{1-\delta}(y_t - y_n) + \phi E_t \pi_{t+1}$$

To account for price shocks in the NKPC, we often write this[10]

$$\pi_t = \frac{\alpha\delta}{1-\delta}(y_t - y_n) + \phi E_t \pi_{t+1} + u_t$$

where u_t is anything that affects marginal cost that's unrelated to the business cycle (e.g., oil shocks, trade unions). To account for interest rate smoothing the second equation is sometimes written

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_n + \gamma(y_t - y_n) + \beta(\pi - \overline{\pi})\right).$$

That is, the optimal interest rate is a weighted average of last period's rate and the optimal one dictated by the standard Taylor rule.

The early eighties in US is a prominent example of these three equations in action. In the early eighties, Paul Volcker, the then FED Chair raised interest rates to almost 20% to restrain inflation, leading to an enormous recession around 1981. Unemployment subsequently rose to about 11% and output fell well below the natural rate; in fact it was the worst U.S. recession since the Great Depression. However, inflation fell from about 10% to 4% by around 1986 and output and unemployment had by then returned to their natural rates so the policy indeed worked. Note, however, that prices were rising all the time, but there was a *moderation* in the rate of wage and price increases.

[10] More generally, all of the equations above should have shock terms, to represent other sources of demand and interest rate changes. The three equation model is really quite simplistic. In reality, there are lags, for example, from monetary policy changes to output, although the model suggests interest rate changes today affect output *today*. Monetary policy, in fact, can take 4 mths to 2 years to have an effect on the real economy.