Class 2

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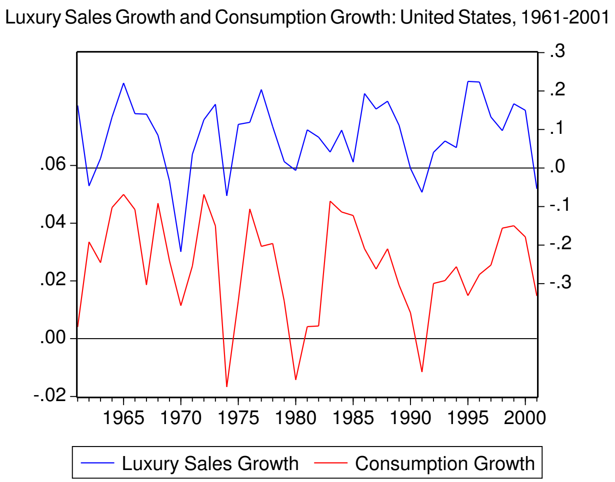
November 23, 2010

Asset Pricing


Luxury Sales Growth and Consumption Growth: United States, 1961-2001
.3
.2
.1
.0
-.1
-.2
-.3
.06
.04
.02
.00
-.02
1965
1970
1975
1980
1985
1990
1995
2000
Luxury Sales Growth
Consumption Growth

$$E(u'(C_{t+1})(1+r_f) = E(u'(C_{t+1})(1+r_{t+1}))$$

$$E\left(u'(C_{t+1})(r_{t+1}-r_f)\right) = 0$$

$$Er_{t+1} = r_f - \frac{Cov(u'(C_{t+1}), r_{t+1})}{E_t(u'(C_{t+1}))} \tag{1}$$

EP Puzzle: volatile u'.
Time Variation in returns relates to time varying covariances (i.e., time-varying risk.) P/D ratio
Cross-Sectional Variation: do some stocks covary more with marginal utility? Such stocks should command higher returns. (Ideally, we can we use this approach to rationalize models such as the Fama-French three factor model.)
Nonlinearity and risk factors: $u' = a + b$

$$u(c) = \frac{c^{1-\theta}}{1-\theta}$$

Habit Persistence

$$u(C_t) = \frac{(C_t - X_t)^{1-\theta}}{1-\theta}$$

$$RA = -\frac{C_t u''}{u'} = \frac{\theta}{S}$$

where

$$S_t = \frac{C_t - X_t}{C_t}$$

Time-varying risk aversion (leading to time varying risk premia, as in the data.)
If expected returns rise in a recession (due to higher risk aversion), prices fall *to generate* those higher returns. So can explain price volatility. (Volatility Puzzle.)
Has large implications for monetary economics too
Tension: IES and savings
Verdelhan and UIP

$$\pi_t = \phi E_t \pi_{t+1} + \zeta(y_t - y_n) + u_t$$

An example: term/risk premia low in last decade on nominal bonds.

One explanation. Recall Phillips curve. Low GDP, low inflation. But with low inflation, expected bond returns fall and bond prices rise. If you are holding a bond, you make capital gain.

So nominal bonds acted as hedge; when GDP is low, you get a capital gain if you hold them. Theory thus predicts that equilibrium returns on nominal bonds are low. (In background, demand for them drives up price, which ultimately lowers returns.)
Deflation, 2008

Application (ignore price variation)

$$E(e_{t+1} - f_t)u'(C_{t+1}) = 0$$

Buy forward today, say $f_t = 100$ and $e_{t+1} = 120$
Call $\gamma = s_{t+1} - f_t$

$$E(XY) = EXEY + Cov(X, Y)$$

$$E(\gamma u') = E\gamma Eu' + Cov(\gamma, u')$$

$$E\gamma = \frac{E(\gamma u')}{Eu'} - \frac{Cov(\gamma, u')}{Eu'}$$

$$E\gamma = -\frac{Cov(\gamma, u')}{Eu'}$$

$$E(e_{t+1} - f_t) = -\frac{Cov(\gamma, u')}{Eu'}$$

$e_t = 10$: 10 euros equals 1 dollar

$$1 + i = \frac{1}{e_t}(1 + i^*)e_{t+1}$$

$$1 + i = \frac{1}{e_t}(1 + i^*)f_t$$

Both have 5 today (both have concave utility)
Next period:
State 1, you have 10, I have 0
State 2, you have 0, I have 10
Formally: we exchange contingent claims. I buy 5 AD (state 2) from you.
You buy 5 AD (state 1) from me.
Market Complete
MU growth equal
Home bias puzzle, consumption correlation puzzle
MBS
Nontraded Goods

Arrow Debreu Securities; all risk can be insured; example, steel (idiosyncratic risk)

Suppose there are two states of the world. In state 1, I get endowment of 1, in state 2 I get 2. Value of endowment is $pY + p_1 Y_1 + 2p_2 Y_2$.

Maximization problem:

$$u(C) + \pi u(C_1) + (1 - \pi)u(C_2) + \lambda(pY + p_1 Y_1 + p_2 Y_2 - pC - p_1 C_1 - p_2 C_2)$$

$$\frac{u'(C_1)}{u'(C)} = \frac{p_1}{p\pi}$$

$$\frac{u'(C_2)}{u'(C)} = \frac{p_2}{p(1-\pi)}$$

Growth rates, not levels

$$u'(C) = \lambda p \tag{2}$$

$$\pi u'(C_1) = \lambda p_1$$

$$\pi u'(C_2) = \lambda p_2$$

ADs can be purchased in Foreign currency for ξ_t and pay off one unit of Foreign currency in period $t+1$ if a given state of nature occurs (which occurs with objective probability π_t).

$$\frac{e_t \xi_t}{p_t} V'(C_t) = \pi_t V'(C_{t+1}) \frac{e_{t+1}}{p_{t+1}} \qquad \Rightarrow \qquad \frac{V'(C_{t+1})}{V'(C_t)} \frac{p_t}{p_{t+1}} \frac{e_{t+1}}{e_t} = \frac{\xi_t}{\pi_t}, \tag{5}$$

that is, the cost of buying the security equals the expected benefit. Meanwhile, for a Foreign consumer buying the security:

$$\frac{\xi_t}{p_t^*} V'(C_t^*) = \pi_t V'(C_{t+1}^*) \frac{1}{p_{t+1}^*} \qquad \Rightarrow \qquad \frac{V'(C_{t+1}^*)}{V'(C_t^*)} \frac{p_t^*}{p_{t+1}^*} = \frac{\xi_t}{\pi_t}. \tag{6}$$

Combining (5) and (6) we have:

$$M_t \frac{p_t}{p_{t+1}} \frac{e_{t+1}}{e_t} = M_t^* \frac{p_t^*}{p_{t+1}^*} \Rightarrow M_t = M_t^* \frac{\frac{e_t p_t^*}{p_t}}{\frac{e_{t+1} p_{t+1}^*}{p_{t+1}}}.$$

Denoting the real exchange rate by $\epsilon_t \equiv \frac{e_t p_t^*}{p_t}$, we have:[1]

$$M_t = M_t^* \frac{\epsilon_t}{\epsilon_{t+1}}$$

[1]Under the assumption of purchasing power parity, we have $p_t = ep_t^* \Rightarrow \epsilon_t = 1$.

$$\Rightarrow \log \frac{\epsilon_{t+1}}{\epsilon_t} = \log M_t^* - \log M_t. \tag{7}$$

And taking unconditional variances gives:

$$Var(\log \frac{\epsilon_{t+1}}{\epsilon_t}) = Var(\log M_t^*) + Var(\log M_t) - 2Cov(\log M_t^*, \log M_t). \tag{8}$$