Class 4

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$e_t = 10$: 10 euros equals 1 dollar
UIP Prediction

$$1 + i = \frac{1}{e_t}(1 + i^*)Ee_{t+1}$$

$$i = i^* + Ee_{t+1} - e_t$$

CIP (this HOLDS, since there is no uncertainty involved)

$$1 + i = \frac{1}{e_t}(1 + i^*)f_t$$

$$i = i^* + f_t - e_t$$

Important point: when f low (i.e., expected strength) i low
Therefore, according to UIP

$$Ee_{t+1} = f_t$$

Puzzle: systematic changes in

$$E(e_{t+1} - f_t)$$

and don't necessarily have $e_{t+1} = f_t$

$$E(e_{t+1} - f_t) = \theta Cov(e_{t+1}, c_{t+1})$$

$$\Rightarrow E(e_{t+1}) = f_t + \theta Cov(e_{t+1}, c_{t+1})$$

$$\Rightarrow E(e_{t+1}) = f_t + RP$$

where RP denotes risk premium. Most importantly (and in contrast to UIP), with a non-zero risk premium

$$E(e_{t+1}) \neq f_t$$

Empirically, when f_t is very low, e_{t+1} is very high. Forward premium puzzle.

Time-varying risk

Need a high $\theta Cov(e_{t+1}, c_{t+1})$ to generate high $E(e_{t+1} - f_t.)$

We want to test if

$$e_{t+1} = f_t$$

Hence

$$e_{t+1} - e_t = f_t - e_t$$

To test theory consider

$$e_{t+1} - e_t = \alpha + \beta(f_t - e_t)$$

and see if $\alpha = 0$ and $\beta = 1$. Large failure of these predictions. (Note that the tests are often in this form since e_{t+1} and f_t are typically *nonstationary*.)
Empirical results: when $f_t - e_t$ (i.e, f_t) is low $e_{t+1} - e_t$ is high
Froot and Thaler, 1990: average from 75 studies is $-.88$

Empirically, when f is lower than usual, e_{t+1} rises
But when f is low, people expect an appreciation and so (from CIP) i is low.
Hence, low interest rates coincide with increase in risk premia

$$e_{t+1} - e_t = \alpha + \beta(f_t - e_t)$$

Using CIP relationship $i = i^* + f_t - e_t \Rightarrow i - i^* = f_t - e_t$, we can rewrite above as

$$e_{t+1} - e_t = \alpha + \beta(i_t - i_t^*)$$

Empirically: when $i_t < i_t^*$ get *weakening.*
Hence, it *appears* you are "better off" investing in a higher interest rate country abroad: you get a higher int rate *and* a currency gain since *that* currency has appreciated relative to yours.
Need to rationalize this. When domestic interest rates are low, people are rewarded well for investing abroad and taking currency risk.

When there's expected appreciation (i.e., low int rates), there is a high risk premium.

$$E(e_{t+1} - f_t) = \theta Cov(e_{t+1}, c_{t+1})$$

Formally, when f is low (i.e., int high) e is high, so covariance positive.
Standard model: $Cov(e_{t+1}, c_{t+1})$ is too small.

Verdelhan (2010, Journal of Finance)
Need a model: when i low (i.e., f low), risk premium is high (i.e., lots of consumption risk.) Why does i low correspond to bad times? Recall that the stochastic discount factor is

$$M_t = \frac{u'(c_{t+1})}{u'(c_t)}$$

The Backus-Smith condition is

$$\log \frac{\epsilon_{t+1}}{\epsilon_t} = \log M_t^* - \log M_t. \quad (1)$$

Cet par, if M_t rises, $\log \frac{\epsilon_{t+1}}{\epsilon_t}$ falls.

In bad times, exchange rate determined by home events, since u' is extremely sensitive to shocks (since consumption close to habit.) M very volatile. In this case, what primarily determines movements in the exchange rate is M. (Little risk for foreigners.)

Hence, if in bad times and $M = \frac{u'(c_{t+1})}{u'(c_t)}$ rises (i.e., c_{t+1} falls), then ϵ_{t+1} falls. That is, if get back domestic shock next period, the exchange rate will *appreciate*.

With habit persistence: in bad times, consumption close to habit, leading to precautionary savings and low interest rates. Risk aversion also rises endogenously.

To summarize: if consumption is close to habit, get low interest rates and a situation where bad shocks cause a currency *appreciation*. This, in turn, makes investment abroad risky, and generates a high premium/expected return.

Atkeson, Alvarez, and Kehoe, AER (2007)

$$\frac{1}{P_t} u'(c_t) = E \frac{(1+i)}{P_{t+1}} u'(c_{t+1})$$

$$\frac{1}{1+i_t} = E \left(\frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}} \right)$$

Pricing kernel in nominal terms

$$-i_t = \log E \left(\frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}} \right)$$

For a log-normally distributed variable

$$\log E(x) = E \log x + \frac{1}{2} Var \log x$$

Hence, assuming the nominal stochastic discount factor is lognormal

$$-i_t = E \log \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}} + \frac{1}{2} var \log \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}}$$

$$\Rightarrow i_t = -E \log \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}} - \frac{1}{2} var \log \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}}$$

Standard New Keynesian models drop latter terms and have

$$i_t = -E \log \frac{u'(c_{t+1})}{u'(c_t)} + \log \frac{1}{\pi_{t+1}}$$

So changes in i affect real economy (i.e., $\frac{u'(c_{t+1})}{u'(c_t)}$) or inflation (i.e., $\frac{1}{\pi_{t+1}}$)

Setting $m_{t+1} = \frac{u'(c_{t+1})}{u'(c_t)} \frac{1}{\pi_{t+1}}$

$$i_t = -E \log m_{t+1} - \frac{1}{2} var \log m_{t+1}$$

For foreign country

$$i_t^* = -E \log m_{t+1}^* - \frac{1}{2} var \log m_{t+1}^*$$

Hence

$$i_t - i_t^* = E \left(\log m_{t+1}^* - \log m_{t+1} \right) - p_t$$

where

$$p_t = \frac{1}{2} \left(var \log m_{t+1}^* - var \log m_{t+1} \right)$$

In real terms

$$\log \frac{\epsilon_{t+1}}{\epsilon_t} = \log M_t^* - \log M_t.$$

Can easily convert to nominal terms

$$\log \frac{e_{t+1}}{e_t} = \log m_t^* - \log m_t.$$

$$i_t - i_t^* = E\left(\log m_{t+1}^* - \log m_{t+1}\right) - p_t$$

$$\log_{t+1} -_t = \log m_t^* - \log m_t.$$

$$i_t - i_t^* = E\left(\log_{t+1} -_t\right) - p_t$$

$$i_t - i_t^* = -p_t$$

$$i_t - i_t^* = \frac{1}{2}\left(var \log m_{t+1} - var \log m_{t+1}^*\right)$$