Debt Sustainability

$$\triangle \frac{D}{Y} = \frac{G-T}{Y} + (r-g)\frac{D}{Y}$$

$$\triangle \frac{D}{Y} = \frac{G-T}{Y} + (i-\pi-g)\frac{D}{Y}$$

For stability, need $\triangle \frac{D}{Y} = 0$

$$\frac{T-G}{Y} = (i-\pi-g)\frac{D}{Y}$$

Low growth (Rogoff)

Now, everything below in *nominal terms*. (No effect on ratios.)

$$\triangle \frac{D}{Y} = \frac{G-T}{Y} + (i - \pi - g)\frac{D}{Y} - \frac{\Delta M}{Y}$$

Seignorage/inflation tax (two roles). Distortionary taxation.

Stabilization Policy

Government, wages, and output

$$\frac{l_{t+1}}{l_t} = \left(\frac{w}{w}\right)^{\frac{1}{\sigma}} = 1 \qquad \Rightarrow l_{t+1} = l_t.$$

$$\frac{l_{t+1}}{l_t} = \left(\frac{(1-\tau)w}{w}\right)^{\frac{1}{\sigma}} \qquad \Rightarrow l_t > l_{t+1}$$

"Another important part of the tax proposal is the moving up to this year of the marginal income tax rate cuts promised in the 2001 law. For example, the reduction in the top marginal rate from 39.6 percent to 35 percent would become fully effective as of January, 2003. Although the cuts in the 2001 law were attractive, the gradual phase-in through 2006 was a bad idea. The prospect of lower future tax rates gives individuals and businesses incentive to defer income and production. For this reason, the rate reductions can actually retard the economy in the short run. In the 1981 law, a similar phase-in of income tax rate cuts likely contributed to the 1982 recession. We learned from the early 1980s that any legislated tax rate changes should occur all at once."

Marginal Tax/Average

Income (wealth) /Substitution (price effect)

E.g., Reduce tax-free income: raise average rate, no effect on marginal rate. Pure income effect.

E.g., Raise tax-free income: raise marginal rate, keep average rate fixed. Pure substitution effect.

Application: Optimal Taxation

Convex costs to labour

Social Planner Problem

Initial Equilibrium is Pareto Optimal (First Welfare Theorem)

Any deviation is suboptimal (ignore externalities etc)

Tax system should get as close as possible to initial equilibrium

Distortionary Taxation

Property Tax

Tax Smoothing (convex costs, “cut rates, widen bands”)

Ramsey Model

Standard Long-Run Model

Solow Model with endogenous savings.

Two key equations: Euler Equation and Capital Accumulation Equation. Related to above, but models dynamics over time.

1 person; $f(k_t) = Ak_t^\alpha$.

$$\sum_{t=0}^{t=\infty} \beta^t u(c_t)$$

$$c_t + s_t = f(k_t)$$

$$k_{t+1} - k_t = i_t$$

Combining

$$k_{t+1} - k_t = f(k_t) - c_t$$

$$c_t = f(k_t) - k_{t+1} + k_t$$

$$\sum_{t=0}^{t=\infty} \beta^t u(f(k_t) - k_{t+1} + k_t)$$

Euler Eqn/Capital Accumulation

Get derivative w.r.t k_{t+1}

$$-\beta^t u'(f(k_t) - k_{t+1} + k_t) +$$

$$\beta^{t+1} u'(f(k_{t+1}) - k_{t+2} + k_{t+1})(1 + f'(k_{t+1})) = 0$$

$$-u'(c_t) + u'(c_{t+1})(1 + f'(k_{t+1})) = 0$$

$$u'(c_t) = \beta(1 + f'(k_{t+1}))u(c_{t+1})$$

Set $u(c) = \log c$ and noting that $\beta = \frac{1}{1+\rho}$

$$\frac{c_{t+1}}{c_t} = \frac{1 + f'(k_{t+1})}{1 + \rho}$$

Two key equations describe the evolution of the economy:

$$\frac{c_{t+1}}{c_t} = \frac{1 + f'(k_{t+1})}{1 + \rho}$$

and

$$k_{t+1} - k_t = f(k_t) - c_t.$$

$$\frac{c_{t+1}}{c_t} = \frac{1 + \alpha A k_{t+1}^{\alpha-1}}{1 + \rho}$$

In steady state, we know consumption will be constant when

$$\alpha A k_{t+1}^{\alpha-1} = \rho$$

$$\Rightarrow k^* = \left(\frac{\alpha A}{\rho}\right)^{\frac{1}{1-\alpha}}$$

Then from the production function, $f(k) = Ak^\alpha$, we can get equilibrium output, $Y = Ak^\alpha = A\left(\frac{\alpha A}{\rho}\right)^{\frac{\alpha}{1-\alpha}}$.

From the capital accumulation equation, we have (since k is constant in steady state)

$$f(k_t) = Ak_t^\alpha = k_{t+1} - k_t + c_t = c_t$$

Capital Taxation

Steady state condition now becomes

$$(1-\tau)\alpha A k_{t+1}^{\alpha-1} = \rho$$

The capital stock then becomes

$$\Rightarrow k^* = \left(\frac{(1-\tau)\alpha A}{\rho}\right)^{\frac{1}{1-\alpha}}$$

If there were l workers (as opposed to one), then the production function would be $Y = Ak^{\alpha}l^{1-\alpha}$. Steady state condition now becomes

$$(1-\tau)\alpha A k^{\alpha-1} l^{1-\alpha} = \rho$$

$$\Rightarrow k^* = \left(\frac{(1-\tau)\alpha A l^{1-\alpha}}{\rho}\right)^{\frac{1}{1-\alpha}} = \left(\frac{(1-\tau)\alpha A}{\rho}\right)^{\frac{1}{1-\alpha}} l$$

$$w = (1-\alpha)Ak^{\alpha}l^{-\alpha}$$

$$w = (1-\alpha)A\left(\frac{(1-\tau)\alpha A}{\rho}\right)^{\frac{\alpha}{1-\alpha}} l^{\alpha}l^{-\alpha}$$

$$\Rightarrow w = (1-\alpha)A\left(\frac{(1-\tau)\alpha A}{\rho}\right)^{\frac{\alpha}{1-\alpha}}$$

Taxation on capital lowers wage (tax incidence)