

# New Goods and Asset Prices

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## Abstract

I propose an extension to the consumption capital asset pricing model to incorporate the introduction of new goods over time and states of nature. In the model, consumers have a love of variety, and consumption consists of different components: product groups and brands. Expected growth of product groups raises expected future marginal utility—thereby increasing the incentive to save and reducing the risk-free rate of interest. Meanwhile, cyclical variation in brand and quality growth makes marginal utility more volatile and countercyclical—thereby raising the expected equity premium. Incorporating the idea into a long-run risk setting has similar implications.

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## Introduction

Because financial assets are ultimately claims to future consumption, new goods should affect consumers' valuation of future payoffs. Moreover, new goods affect marginal utility in different ways: the release of a new drug, for instance, has a different effect to that of a new cereal. To address these issues, this paper proposes an extension to the consumption capital asset pricing model (CCAPM) to incorporate changes in product variety over time and states of nature.

By treating all goods—such as cars and wine—as perfect substitutes, the CCAPM implies variation in consumption expenditure fully determines variation in marginal utility. And essentially this is the source of the equity premium puzzle: expenditure growth is relatively smooth. As a result, there is little risk to holding equity, so only a high level of risk aversion can justify a large equity premium. Closely related to this is the risk-free rate puzzle: faced with higher expected consumption of the *same* good in the future, coupled with diminishing marginal utility to *that* good, why do consumers still save, thus keeping the risk-free rate so low?

People in my setting have a love of variety and consume an increasing number of goods. By variety, I mean distinct product groups—such as cars—and relatively similar brands within each group—such as Volvos and Fords. Central to the model is variety growth: the number of goods, along with their quality, varies over time and states of nature. In my baseline model, I maintain the basic framework of time-separable, expected power utility, with complete, frictionless markets.

By purchasing financial assets, consumers smooth marginal utility over time and states of nature. But because they now care about *what* they consume, changes in product variety result in different “states of the world,” with states described by both the level of consumption expenditure *and* the variety of goods available. As we shall see, product group growth slows the decline in marginal utility over time, while cyclical variation in brand and quality growth raises the level of consumption risk. These dynamics affect asset prices.

To see the idea, consider the “tree economy” setting of [Lucas \(1978\)](#), where trees bear fruit—say, oranges—randomly. Faced with orange growth over time, consumers desire to borrow, raising the risk-free rate. But in my setting the future brings not only more oranges but also new kinds of fruit: looking ahead, trees also bear apples, grapes, pears, and so on. This increases the demand for future consumption and raises savings. The risk-free rate falls.

Turning to the equity premium, in Lucas’s setting, the covariance of marginal utility with the number of oranges from a tree determines the price of the tree. Trees bearing more oranges in a boom provide poor insurance and therefore command a risk premium. Yet along with more oranges here comes greater orange *variety*: now there are Florida oranges, sweet ones, organic ones, and so on. By reducing the effective price of oranges, greater variety raises the covariance between consumption *services* and equity returns. The equity premium rises.

Although product variety clearly grows over time, is it procyclical? A number of models make this prediction. Chief among these are [Shleifer \(1986\)](#), who models firms introducing new products at the same time, causing “implementation cycles”; [Schmookler \(1966\)](#), who argues that greater demand in booms raises innovation; and, most famously, [Schumpeter \(1939\)](#), who describes the business cycle in terms of “waves of creative destruction.” More recently, several papers explore the role of firm entry and expanding product variety over the business cycle (see e.g., [Melitz, Ghironi, and Bilbiie, 2006](#); [Bergin and Corsetti, 2005](#)). According to these models, procyclical profits induce firm entry, raising variety growth. And consistent with these predictions, procyclical variety growth is a feature of the data (see e.g., [Broda and Weinstein, 2010](#); [Axarloglou, 2003](#)). [Broda and Weinstein](#) find “net product creation is strongly procyclical,” while [Melitz et al.](#) report that U.S. firms introduce more varieties during expansions. Linked to this, [Broda and Weinstein](#) argue that inadequate treatment of new goods in price indices leads to “substantial bias”. For consumption-based asset pricing, this issue is potentially significant.<sup>1</sup>

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<sup>1</sup>Some recent contributions analyze the CCAPM in a multi-good framework, but restrict their analy-

I proceed as follows. After outlining the baseline model in Section 1, I describe the asset pricing implications in Section 2. Incorporating new goods into a price index, I show marginal utility now depends on the variety of goods consumed. In the baseline model, product group growth raises marginal utility, while brand and quality growth reduce it; intuitively, group growth creates more “needs,” while brand and quality growth satiate them. The model identifies four risk premia: one arising from consumption expenditure risk and three from composition risk—that is, risk related to variation in brand, quality, and group growth. Meanwhile, the expected risk-free rate depends on the long-run trend in variety growth, along with precautionary savings.

Having presented the theory, I next survey empirical evidence on variety growth in Section 3. Using theory together with annual trademark data, I approximate the annual degree of cyclical variation in brand, quality, and group growth, along with the associated welfare gains. Following this, I examine implications for inflation mismeasurement in Section 4. I then calibrate the expected equity premium and risk-free rate for a range of parameter values in Section 5. Compared to the standard CCAPM, the expected equity premium is higher, and the risk-free rate is lower. In Section 6 I extend the framework to incorporate recursive preferences and long-run risk in variety growth; given the degree of path dependence in innovation, a long-run risk framework provides a natural setting to model asset prices. In contrast to power utility, this setup generates plausible variation in the price-dividend ratio. In Section 7 I show that variety growth can explain a cross-sectional spread of returns. Finally, Section 8 concludes.

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sis to variations in consumption over a *fixed* product space. For example, [Piazzesi, Schneider, and Tuzel \(2007\)](#) incorporate a consumption bundle of non-housing consumption and housing services into the CCAPM. They derive an associated price index, and show that cyclical variation in the housing share raises the expected equity premium, while long-run trends and volatility in the housing share reduce the risk-free rate. Focussing on a consumption bundle comprising nondurable and durable components, [Pakoš \(2004\)](#) and [Yogo \(2006\)](#) highlight how the interaction of both components can raise aggregate consumption risk. Examining the role of consumption commitments, [Chetty and Szeidl \(2007\)](#) show how commitments increase the variability of discretionary consumption and, in turn, consumption risk. Closer to this paper, [Ait-Sahalia, Parker, and Yogo \(2004\)](#) introduce luxury goods into the CCAPM, and show how the covariance of luxury goods consumption with equity returns raises the equity premium. Although the classic contributions of [Merton \(1973\)](#) and [Breedon \(1979\)](#) analyze variation in consumption opportunities, they focus on changes in the relative prices of *existing* products.

# 1 The Benchmark Model: Power Utility

## 1.1 The Economic Environment

There is a representative consumer who lives for  $T$  periods. The consumer has preferences over a continuum of existing and potential products, including leisure. There is latent demand for all potential goods, and no good is essential. The consumer receives income from investment returns and labor supply.

Consumption consists of different components, and there are two margins of differentiation: product groups and brands. Product groups represent broad categories of goods without close substitutes, for which demand is relatively inelastic; for example, new medicines, cars, microwave ovens, cell phones, and so on. There is a continuum of differentiated product groups, indexed along the interval  $(0, \infty)$ , but at any time  $t$ , only a measure  $n_t < \infty$  is available for purchase. There is a common elasticity of substitution between all groups, and since groups are imperfect substitutes, this is less than one. New goods on this margin represent “breakthrough” innovations. Broadly, these goods correspond to the approximately 200 product categories in the Consumer Price Index (CPI), and growth on this “extensive” margin will raise marginal utility, increase savings, and reduce the risk-free rate.

Associated with each group is a continuum of brands, also indexed on  $(0, \infty)$ . Brands represent different varieties of goods within a given group—in terms of attributes like style, flavor, size, and so on. At time  $t$ , there is a measure  $m_t < \infty$  of brands available in each group.<sup>2</sup> Because brands are relatively good substitutes, demand on this margin is elastic, and the common elasticity of substitution between brands exceeds one; new goods on this margin represent minor innovations. In the context of the CPI, these goods correspond to the products within the individual categories. Variation on this “intensive” margin satiates consumer needs and exhibits high frequency variation; as such, any procyclical variation will affect the equity premium. In addition to changes

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<sup>2</sup>For convenience, I ignore indivisibilities and the non-integral nature of the variables  $n$  and  $m$ , and from now on refer to them loosely as numbers.

in the number of brands, there are also quality improvements to each existing brand over time; at time  $t$ , all existing brands are of uniform quality  $A_t \in (0, \infty)$ .

I index groups by  $j \in [0, n_t]$  and brands by  $i \in [0, m_t]$ . Thus  $c_{jt}$  denotes the consumption service flow from group  $j$  at time  $t$ , while  $c_{jit}$  denotes the consumption of brand  $i$  from group  $j$  at time  $t$ . All existing goods have equal prices, while non-existent goods have infinite prices. The number of brands  $m$ , the number of product groups  $n$ , the level of quality  $A$ , wages  $w$ , and consumption expenditure  $C$ , exhibit lognormal growth and evolve according to the general process

$$g_{y,t+1} = \mu_y + \sigma_y \epsilon_{y,t+1},$$

where  $g_{y,t+1} \equiv \log y_{t+1} - \log y_t$ , and  $\epsilon_{y,t+1} \sim i.i.d. N(0, 1)$  for all  $y \in m, A, n, w, C$ .

The consumer has a love of variety for groups and brands. For groups, one can explain this by the welfare improvement associated with major innovations: new groups satisfy previously unmet needs. For brands, one can explain a love of variety in two ways. First, with diminishing marginal utility to each brand, there is a welfare gain to smoothing consumption over more brands. Confronted with a variety of yoghurts, for example, the consumer might prefer to consume a little of each flavor; this is the [Dixit and Stiglitz \(1977\)](#) setup. Second, greater variety enables consumers to attain their ideal brand or bundle of characteristics; this is the [Lancaster \(1979\)](#) formulation. Either way, consumer welfare rises as the number of brands increases. For simplicity, I use the more tractable Dixit-Stiglitz setup.

## 1.2 Consumer Preferences

Utility each period derives from consumption services from different product groups,  $c_{jt}$ , and labor supply  $l_t$ . Consumption services from a group are given by the constant elasticity of substitution index:

$$c_{j_t} \equiv m_t^{v+1-\frac{1}{\alpha}} \left( \int_0^{m_t} (A_t^\gamma c_{j_{i_t}})^\alpha di \right)^{\frac{1}{\alpha}}, \quad (1)$$

where  $\alpha \in (0, 1)$ ,  $c_{j_{i_t}} \geq 0$  refers to the amount of brand  $i$  consumed, and  $m_t \in (0, \infty)$  denotes the number of brands consumed.<sup>3</sup> A rise in quality  $A_t$  increases the utility derived from the consumption of each brand, and  $\gamma > 0$  mediates the taste for quality. Since there is a continuum of brands, the elasticity of substitution between brands within each group is  $\frac{1}{1-\alpha} \in (1, \infty)$ . Associated with group  $c_{j_t}$  is the price index

$$p_{j_t} = A_t^{-\gamma} m_t^{-v-1+\frac{1}{\alpha}} \left( \int_0^{m_t} p_{j_{i_t}}^{\frac{\alpha}{\alpha-1}} di \right)^{\frac{\alpha-1}{\alpha}}, \quad (2)$$

where  $p_{j_{i_t}}$  denotes the price of brand  $c_{j_{i_t}}$  at time  $t$ .

Following [Benassy \(1996\)](#),  $v \in [0, \infty)$  mediates the taste for brand variety, and governs the elasticity of the marginal utility of consumption with respect to the number of brands consumed. This parameter disentangles the distinct concepts of elasticity of substitution between brands—which also equals the elasticity of demand for each brand—and love of variety.<sup>4</sup> As a result, this formulation can handle situations where the consumer might be highly responsive to price changes, but still has a large taste for variety; or cases where the consumer has little taste for variety, but perceives goods as imperfect substitutes.

The utility from consumption services  $c_{j_t}$  is given by

$$u(c_{j_t}) - u(0) = \frac{(c_{j_t} + \epsilon)^{1-\theta}}{1-\theta} - \frac{\epsilon^{1-\theta}}{1-\theta} \geq 0,$$

where  $\theta > 1$ ,  $\epsilon > 0$ , and  $c_{j_t} \gg \epsilon \approx 0$ . The constant  $\epsilon$  is arbitrarily small and governs the

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<sup>3</sup>In equilibrium this will equal the number of brands available. Given symmetry and strict concavity, the consumer purchases all available brands in equal quantities. For simplicity, I make no distinction between the number of brands available and the number consumed.

<sup>4</sup>To see why, suppose consumption expenditure on a group is  $C_t$  and all goods have a price of one. In equilibrium  $c_{j_t} = m_t^{v+1-\frac{1}{\alpha}} m_t^{\frac{1}{\alpha}-1} A_t^\gamma C_t = m_t^v A_t^\gamma C_t$ . The parameter  $v \geq 0$  now mediates the marginal utility gain to consuming additional brands. By comparison, the standard [Dixit and Stiglitz](#) function conflates the degree of love of variety with the elasticity of substitution and implicitly assumes  $v = \frac{1}{\alpha} - 1$ .

utility gain from consuming a positive quantity of the group. Since  $\epsilon \approx 0$ , even a small amount of consumption on a new group raises utility significantly. That is, there is a sizable welfare gain to consuming distinct new innovations—e.g., a car—irrespective of the quantity consumed. Technically, since  $\theta > 1$ ,  $\epsilon$  also ensures the utility flow from consuming a group is positive, and utility is well-defined when  $c_{j_t} = 0$ .

Period utility from consumption services when the consumer purchases  $n_t$  groups is

$$n_t^\phi \int_0^{n_t} \frac{(c_{j_t} + \epsilon)^{1-\theta}}{1-\theta} - \frac{\epsilon^{1-\theta}}{1-\theta} dj, \quad (3)$$

where  $\phi > -1$ . The constant  $\frac{1}{\theta} < 1$  is the elasticity of intertemporal substitution of consumption services from each group across time.<sup>5</sup> Yet because groups are separable in utility,  $\frac{1}{\theta}$  is also the elasticity of substitution between groups. As a result, consumption services in different periods and consumption services of different groups are equally substitutable. Because groups are imperfect substitutes, while much empirical evidence indicates the intertemporal elasticity of substitution is below one, this is a reasonable simplification.

Here,  $\phi$  plays a role similar to that of  $v$  in the discussion of brands: it disentangles the degree of love of variety for groups from the elasticity of substitution between groups. To see this, let  $\bar{c}_{j_t}$  denote the equilibrium level of consumption services in each group at time  $t$ . If the consumer purchases  $n_t$  product groups, then by symmetry the equilibrium level of utility from consumption in period  $t$  is given by  $n_t^{\phi+1} (u(\bar{c}_{j_t}) - u(0))$ . Holding the level of consumption services in each group constant at  $\bar{c}_{j_t}$ ,  $\phi > -1$  now mediates the marginal utility gain to consuming additional groups; the restriction,  $\phi > -1$ , ensures that utility in this case is increasing in the number of groups consumed.

Holding  $c_{j_t}$  on all groups fixed, if  $-1 < \phi < 0$ , there is decreasing marginal utility to

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<sup>5</sup>Strictly speaking, this is an approximation and is only true as  $c_{j_t} \rightarrow \infty$ . The elasticity of intertemporal substitution (IES) of  $c_{j_t}$  is  $\frac{c_{j_t} + \epsilon}{\theta c_{j_t}}$ , but since  $\epsilon \approx 0$ ,  $\frac{c_{j_t} + \epsilon}{\theta c_{j_t}} \approx \frac{1}{\theta}$ .

the number of groups consumed. When  $\phi = 0$ , groups are independent and additively separable in utility. In contrast, if  $\phi > 0$ , there is increasing marginal utility to the number of groups consumed, reflecting a strong love of variety: new groups now raise the marginal utility gain from existing ones. As an example, suppose there are only two groups—food and recreation—and a firm introduces a third group—say, cars. Keeping expenditure on all groups fixed, the introduction of cars has two effects. First, because the consumer can now travel to more restaurants, the marginal utility of food rises. Second, since the consumer can now travel easily to airports for vacations, the marginal utility of recreation also rises. This way, the new group raises the marginal utility of existing groups.

### 1.2.1 Specification of Lifetime Utility

Taking the distributions of  $m_t$ ,  $A_t$ ,  $n_t$ , and  $w_t$  as given, the consumer maximizes expected lifetime utility:

$$\mathbb{U} = \mathbb{E}_0 \sum_{t=0}^T \beta^t \left( n_t^\phi \int_0^{n_t} u(c_{jt}) - u(0) dj - h(l_t) \right), \quad (4)$$

where  $\mathbb{E}_0\{ \}$  is an expectations operator, conditional on information at time  $t = 0$ , and  $h''(l_t) > 0$ . The subjective rate of time preference is  $\rho > 0$ , and  $\beta \equiv \frac{1}{1+\delta}$ . The function  $h(l_t)$  captures the disutility of supplying labor  $l_t$ .

## 1.3 The Solution

Faced with an asset paying a nominal return  $i_{t+1}$  in period  $t + 1$ , the Euler equation for the optimal evolution of consumption for each product group  $c_j$  is

$$\beta \mathbb{E}_t \left[ (1 + i_{t+1}) \left( \frac{p_{jt}}{p_{j,t+1}} \right) \left( \frac{u'(c_{j,t+1})}{u'(c_{jt})} \right) \left( \frac{n_{t+1}}{n_t} \right)^\phi \right] = 1, \quad (5)$$

for  $j = 1 \cdots n$ .

Because of diminishing marginal utility to brands, it is optimal to consume an equal

quantity of each. Assuming a price of  $p_t$  for each existing good at time  $t$ , the quantity demanded of each existing good is then  $c_{ji_t} = \frac{C_t}{p_t m_t n_t}$  for all  $j \in [0, n_t]$  and  $i \in [0, m_t]$ , where  $C$  corresponds to consumption expenditure; given equal prices and the significant welfare gain to consuming a new product group, I assume it is optimal to consume all available groups<sup>6</sup> Plugging these demands into (1) gives the optimal level of consumption services from group  $j$  at time  $t$ :  $m_t^v A_t^\gamma \frac{C_t}{p_t n_t}$ . Setting  $\epsilon = 0$  for simplicity, imposing symmetry, substituting this into the Euler equation (5) above gives

$$\beta \mathbb{E} \left[ (1 + r_{t+1}) \left( \frac{c_{t+1}}{c_t} \right)^{-\theta} \left( \frac{m_{t+1}}{m_t} \right)^{v(1-\theta)} \left( \frac{A_{t+1}}{A_t} \right)^{\gamma(1-\theta)} \left( \frac{n_{t+1}}{n_t} \right)^\zeta \right] = 1, \quad (6)$$

where  $\zeta = \phi + \theta$ , and  $r$  corresponds to the real return on an asset.<sup>7</sup> Absent variety growth, this reduces to the standard Euler equation.

## 1.4 Discussion of Intertemporal Problem

Setting  $p_t = 1$  for simplicity and substituting  $c_{ji_t} = m_t^v A_t^\gamma \frac{C_t}{n_t}$  into (4), imposing symmetry, and assuming  $\epsilon = 0$  gives the period indirect utility function:<sup>8</sup>

$$V(C_t, m_t, n_t, A_t) \approx \frac{n_t^\zeta m_t^{v(1-\theta)} A_t^{\gamma(1-\theta)} C_t^{1-\theta}}{1-\theta} + \frac{n_t^{\phi+1}}{(\theta-1)\epsilon^{\theta-1}}. \quad (7)$$

In contrast to the standard model—where  $V(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$ —the indirect utility function (7) now depends on the variety of goods consumed. By setting  $n_t = m_t = A_t = 1$ , the resulting utility function represents the same preferences as the standard one-good model. Also, the intertemporal elasticity of substitution for real consumption expenditure is  $\frac{1}{\theta}$ . For a given level of variety, a high  $\theta$  implies sharp diminishing marginal

<sup>6</sup>Footnote 8 presents a condition ensuring this holds.

<sup>7</sup>I define the gross real return is  $1 + r_{t+1} = \left( \frac{p_t}{p_{t+1}} \right) 1 + i_{t+1}$ , where  $p_t$  denotes the price of each good at time  $t$ .

<sup>8</sup>Provided the parameter restriction,  $\epsilon < \left( \frac{\phi+\theta}{\phi+1} \right)^{\frac{1}{1-\theta}} \left( \frac{C_t m_t^v A_t^\gamma}{p_t} \right)$ , holds, indirect utility is increasing in the number of groups consumed, and hence consumers purchase all groups. Because  $\epsilon$  is infinitesimally small by assumption, I assume it always satisfies this condition.

utility to consumption expenditure at any given point in time. Yet because of product variety growth, marginal utility does not necessarily fall this fast *over* time. Later on, variety growth plays an important role in reconciling a low intertemporal elasticity of substitution at any given point in time with apparently high intertemporal substitution over time.

## 1.5 The Effect of Variety on Marginal Utility

According to Eq. (7), marginal utility is increasing in the number of groups, but decreasing in the number of brands and level of quality. To see why, recall that the optimal value of  $c_{jt}$  is  $m_t^\nu A_t^\gamma \frac{C_t}{n_t}$ . Expressed this way, one can view  $\frac{C_t}{n_t}$ ,  $A_t$ , and  $m_t$  as distinct inputs, all combining to produce consumption services  $c_{jt}$ . Because these inputs enter in Cobb-Douglas form, the elasticity of substitution between them is one.

For expenditure allocation, the consumer has preferences defined over two margins. The first is the intensive margin; i.e., the allocation of inputs,  $m_t$ ,  $A_t$ , and  $\frac{C_t}{n_t}$ , to produce consumption services  $c_{jt}$  from each group. The second is the extensive margin: the allocation of consumption services from each group,  $c_{jt}$ , across time. Given  $\theta > 1$ , the elasticity of substitution between  $m_t$ ,  $A_t$ , and  $\frac{C_t}{n_t}$ —i.e., one—exceeds the elasticity of substitution of consumption services across time and groups,  $\frac{1}{\theta}$ . Compared to the level of consumption services across time,  $m_t$ ,  $A_t$ , and  $\frac{C_t}{n_t}$  are therefore relatively good substitutes within each period.

By raising the level of consumption services derived from a given amount of expenditure on each existing group, increases in  $m_t$  and  $A_t$  in a period lead to consumption “deepening.” And since the consumer cares more about smoothing the level of  $c_{jt}$  across time, they quickly become satiated as  $c_{jt}$  rises for each  $j$  in a given period. Intuitively, since  $m_t$ ,  $A_t$ , and  $\frac{C_t}{n_t}$  are relatively good substitutes, increases in  $m_t$  or  $A_t$  act as substitutes for consumption expenditure  $C_t$ , and this satiates the consumer. Rather than consuming more in a given period, the consumer seeks to shift real consumption resources  $C_t$  to other periods. That is, a rise in  $m_t$  or  $A_t$  in a period reduces marginal

utility that period.

A rise in the number of groups  $n_t$  in a period has two effects on marginal utility. First, since the consumer smooths expenditure  $C_t$  over groups, a rise in  $n_t$  reduces the consumption of each group. Because the consumer has a relatively strong desire to smooth the levels of  $c_{jt}$  across time, this consumption “widening” raises the marginal utility of consumption for each group in period  $t$ . Second, there is a direct effect due to  $\phi$ , capturing the degree of complementarity between groups: holding expenditure on all groups fixed, increasing the number of groups consumed affects the marginal utility of existing groups. Summarizing, we have the following propositions:

**Proposition 1** : *A rise in the number of brands  $m_t$  or the level of quality  $A_t$  reduces the marginal utility of consumption; i.e.,  $\frac{\partial^2 V}{\partial m_t \partial C_t} < 0$ , and  $\frac{\partial^2 V}{\partial A_t \partial C_t} < 0$ .*

**Proposition 2** : *A rise in the number of groups  $n_t$  raises the marginal utility of consumption; i.e.,  $\frac{\partial^2 V}{\partial n_t \partial C_t} > 0$ .*

## 2 Asset Pricing and Equilibrium Returns

In this section I present the asset pricing implications of the model. Regarding notation,  $\sigma_x$  denotes the standard deviation of the growth of the random variable  $x$ ,  $\rho_{x,y}$  the correlation of random variables  $x$  and  $y$ , and  $\sigma_{x,y}$  their covariance;  $r_f$  denotes the return on the risk-free asset.

### 2.1 Risk-Free Asset Return

To obtain an expression for the expected risk-free rate, I set  $r_t = r_{ft}$  in (6) above, and assume consumption, variety, and returns are homoskedastic and jointly lognormally distributed. Dropping time subscripts, the expected log risk-free rate reduces to

$$\begin{aligned}
r_f = & \underbrace{\delta + \theta\mu_c}_{CCAPM} - v(1 - \theta)\mu_m - \gamma(1 - \theta)\mu_A - \zeta\mu_n - \underbrace{\frac{1}{2}(\theta^2\sigma_c^2 + (v(1 - \theta))^2\sigma_m^2)}_{CCAPM} \\
& + (\gamma(1 - \theta))^2\sigma_A^2 + \zeta^2\sigma_n^2 + v\theta(1 - \theta)\sigma_{m,c} + \gamma\theta(1 - \theta)\sigma_{A,c} \\
& + \theta\zeta\sigma_{n,c} - v\gamma(1 - \theta)^2\sigma_{A,m} - v\zeta(1 - \theta)\sigma_{m,n} - \gamma\zeta(1 - \theta)\sigma_{A,n}. \tag{8}
\end{aligned}$$

By reducing expected future marginal utility, trend growth in quality, the number of brands, and level of consumption reduces the incentive to save and raises the expected risk-free rate. By contrast, by increasing expected future marginal utility, expected group growth raises savings and lowers the risk-free rate. Volatility in consumption and variety growth raises precautionary savings, again reducing the risk-free rate. Because consumption, brand, and quality growth all have the same qualitative effect on marginal utility, positive covariance between them amplifies variation in marginal utility growth. This increases the need for precautionary savings and lowers the risk-free rate. Yet because group and each of brand and quality growth have opposing effects on marginal utility, negative covariance between these variables reduces variation in marginal utility. This reduces the need for precautionary savings and raises the risk-free rate.

The model nests the standard CCAPM: absent a love of variety, only the CCAPM terms remain. With observed consumption growth of around 2 percent,  $\theta = 5$ , and a rate of time preference of  $\delta$  of one percent, the CCAPM predicts a risk-free rate of around 11 percent. Yet empirically,  $r_f \approx 1$  percent. This inability of the standard CCAPM to reconcile a steep consumption profile with a strong desire to smooth consumption is the risk-free rate puzzle.

### 2.1.1 New Expression for the Risk-Free Rate with Variety Growth

Denoting the real wage at time  $t$  by  $w_t$ , the standard labor optimality condition is

$$w_t V'(C_t) = h'(l_t). \quad (9)$$

Over long periods of time, labor hours per capita are approximately constant (see e.g., Ramey and Francis, 2009). To reconcile this observation with the model, the rate of increase of the wage  $w_t$  must equal the rate of decline in marginal utility  $V'(C_t)$ . Substituting the model's expression for marginal utility into Eq.(9), and then taking growth rates and means then implies

$$-\theta\mu_c + \underbrace{v(1-\theta)\mu_m + \gamma(1-\theta)\mu_A + \zeta\mu_n}_{\approx(\theta-1)\mu_c > 0} = -\mu_w. \quad (10)$$

According to the standard model, where  $\mu_m = \mu_A = \mu_n = 0$ , marginal utility falls on average at a rate of  $\theta\mu_c$ . Augmenting the model with new goods introduces the extra terms in underbraces in Eq.(10), which can reconcile the model with stable labor hours without imposing  $\theta = 1$ . Noting  $\mu_w \approx \mu_c$  in the long run, the additional terms imply that the rate of decline in marginal utility over time is lower relative to the standard model. Substituting (10) into (8) and assuming  $\mu_w = \mu_c$  gives a simplified expression for the expected risk-free rate, which I use to calibrate the model:

$$\begin{aligned} r_f = & \delta + \mu_c - \underbrace{\frac{1}{2}(\theta^2\sigma_c^2 + (v(1-\theta))^2\sigma_m^2 + (\gamma(1-\theta))^2\sigma_A^2 + \zeta^2\sigma_n^2)}_{CCAPM} + \\ & v\theta(1-\theta)\sigma_{m,c} + \gamma\theta(1-\theta)\sigma_{A,c} + \theta\zeta\sigma_{n,c} - v\gamma(1-\theta)^2\sigma_{A,m} \\ & - v\zeta(1-\theta)\sigma_{m,n} - \gamma\zeta(1-\theta)\sigma_{A,n}. \end{aligned} \quad (11)$$

According to the labor evidence, variety growth moderates the decline in marginal utility over the long run. In a deterministic setting, this causes savings to rise, causing a decline in the risk-free rate. Whether the risk-free rate in (11) rises or falls relative to the standard model depends on the taste parameters and the magnitudes of the trend growth and covariance terms.

### 2.1.2 Risky Asset Return

Taking differences and unconditional expectations of the linearized Euler equations for both the risk-free and risky assets gives the expected excess log returns

$$\mathbb{E}(r - r_f) + \frac{\sigma_r^2}{2} = \underbrace{\theta \sigma_{r,c}}_{CCAPM} + v(\theta - 1)\sigma_{r,m} + \gamma(\theta - 1)\sigma_{r,A} - \zeta\sigma_{r,n}. \quad (12)$$

Given  $\theta > 1$ , brand and quality growth satiate the consumer. For this reason, their positive covariance with returns raises the level of consumption risk. To compensate investors for this risk, there are two additional risk premia: one for quality variation and another for brand variation. As a result, fluctuations in brands and quality lead to an equity premium even if consumption expenditure is stable. By contrast, positive covariance of the asset return with group growth provides consumption insurance—reducing the expected return. Thus while the nonstationarity of  $m$  and  $A$  generate an equity premium at all horizons, it is an empirical matter whether the premium rises or falls overall. The taste parameters  $v$ ,  $\gamma$ , and  $\zeta$  govern the size of the risk premia.

The equity premium puzzle refers to the failure of the CCAPM term to match the equity premium. For example, in the United States over the twentieth century, the left-hand side has been around 6 percent, while  $\sigma_{r,g_c} \approx .1$ .<sup>9</sup> Since the quantity of consumption risk, as captured by  $\sigma_{r,g_c}$ , is relatively low, the level of risk aversion must be high to justify the large equity premium; here we need  $\theta \approx 60$ .

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<sup>9</sup>Here, the S & P index proxies for the market portfolio, while Treasury bills proxy for the risk-free asset. Consumption growth refers to the growth of consumption nondurables and services expenditure. Throughout, I use data on consumption expenditure and price deflators from the National Income and Product Accounts (NIPA), provided by the BEA. I use the beginning-of-period time convention in the calculation of growth rates; that is, I assume all reported consumption takes place at the beginning of each period. To obtain per capita values, I deflate all the consumption series by population data, available from the U.S. Census Bureau.

### 3 Empirical Evidence on Variety Growth

To determine expected returns in this multi-good setup, it is important to address two questions. First, how does product variety change over time and over the business cycle? Second, how do consumers value these changes? Although the welfare gains to product creation have been examined elsewhere (see e.g., [Hausman, 1997](#); [Nevo, 2001](#)), less attention has been paid to trends and fluctuations in variety growth.

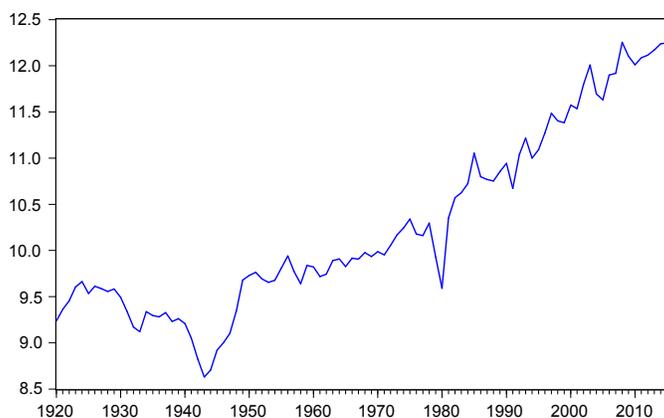
#### 3.1 Evidence on Variety Growth

[Klenow \(2003\)](#) documents that the number of products carried by an average supermarket rose from 10,425 in 1978 to 40,333 in 2000—yielding an annual growth rate of 6.7 percent. [Ebert and Giffen \(2006\)](#) report that firms in the United States introduce more than 25,000 new household, grocery, and drugstore items annually. [Mahajan and Wind \(1992\)](#) find that 25 percent of corporations' revenues come from products introduced in the past 3 years. Additional evidence of product creation comes from new trademark growth, which has averaged over 3.3 percent a year between 1927 and 2016; [Figure 5](#) depicts the steady rise in new trademark registration over time. Analyzing quality growth, [Bils \(2004\)](#) show that it has averaged at least 5.8 percent a year for durable goods—more than double the rate implied by the CPI.

Consumption patterns reflect these changes in product variety. Using a database that covers around 40 percent of all goods in the CPI—mainly in grocery, drugstore, and mass-merchandise sectors—between 1994–2003, [Broda and Weinstein \(2010\)](#) find that the value of new goods represents around 9 percent of annual consumption expenditure. Furthermore, 40 percent of household expenditure in a given year is on goods that were created in the last four years, while 64 percent is on goods introduced in the last 9 years. Even in the face of higher relative prices, [Bils and Klenow \(2001b\)](#) report that consumption shifts towards those product categories with more product innovation—suggesting the welfare gains to new goods outstrip the associated price

increases. This evidence suggests significant changes in consumption composition over time.

Especially relevant here is evidence on the cyclical nature of variety growth (see e.g., Broda and Weinstein, 2010; Axarloglou, 2003; Divinney, 1990). Using barcode data, Broda and Weinstein provide extensive evidence that net product creation is “strongly procyclical.” Figure 5 shows the cyclical nature of new trademark registrations in the United States between 1920 and 2016. Mintel (2009), a marketing agency, reports that the number of new product launches from U.S. food and beverage producers fell 51 percent in the first quarter of 2009 from a year earlier. Figure 2 shows how the onset of recession in the United States in 2008 was associated with a sharp decline in new product introduction. On average it took 8 years for the level of product introductions to revert to its 2008 level. The procyclical nature of firm entry, imports, and new trademark registration provides additional evidence of procyclical variety growth (Yorukoglu, 2000; Bergin and Corsetti, 2005; Ghironi and Melitz, 2007).



**Figure 1: NEW TRADEMARKS REGISTERED IN UNITED STATES (LOG SCALE), 1920-2016.**  
SOURCE: HISTORICAL STATISTICS OF THE UNITED STATES.

### 3.1.1 The Cyclical Nature of Variety Growth

Most of the existing evidence of procyclical variety growth comes from goods—such as new food products and trademarks—that almost certainly constitute brands and

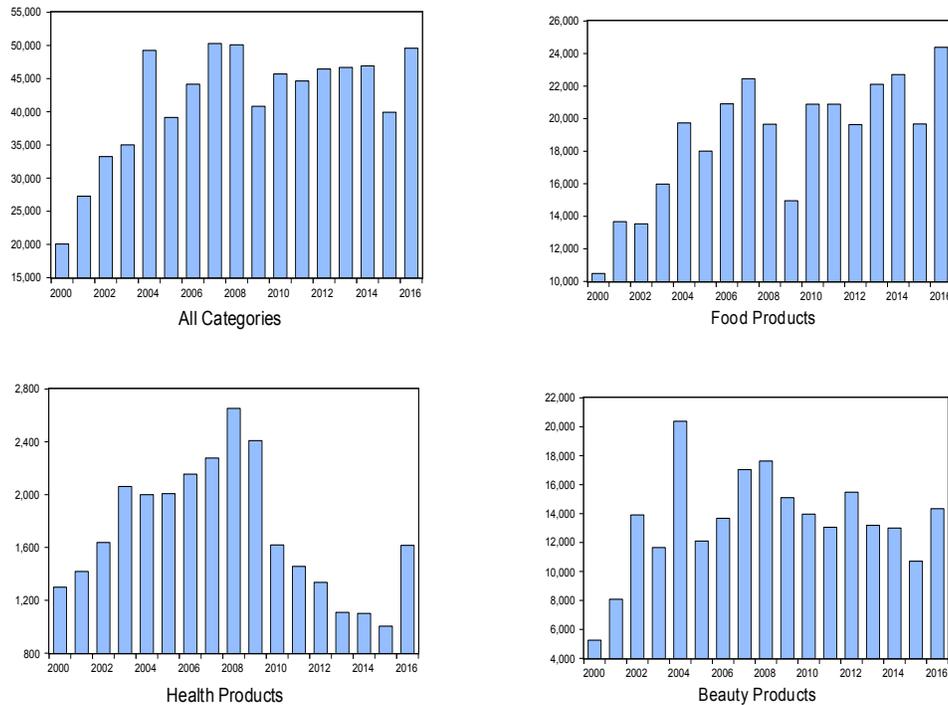


Figure 2: NEWLY INTRODUCED PRODUCTS IN U.S., 2000-2016  
 Source: Mintel GNPD Database.

relatively minor innovations. By contrast, the limited available evidence suggests product group growth is acyclical. [Oi \(1997\)](#) surveys evidence indicating the introduction of “major innovations”—i.e., product groups—bears little relationship to the business cycle, while the introduction of “improvement innovations” does. He attributes this to the nature of product groups: because demand for product groups is inelastic, their sales are insensitive to business cycle conditions. Indeed, [Mensch \(1978\)](#) and [Kleinknecht \(1981\)](#) present evidence that firms introduce “basic innovations”—corresponding to product groups—during recessions. According to them, low demand for existing products in a recession compels firms to innovate to generate new sources of revenue.

A feature of product groups is their development can take decades, and firms often telegraph the arrival of such products years in advance. According to [Agarwal and Bayus \(2002\)](#), the radio, for example, was invented in 1912 and commercialised in 1919;

the electric razor was invented in 1928 and commercialised in 1937; while the clothes dryer was invented in 1930 and commercialized in 1935. Related to this is the existence of a product life-cycle for a group: initially producers target high-income groups, but as costs fall and competition intensifies, they target the broader market. [Tellis, Stremersch, and Yin \(2003\)](#) examine the diffusion of product groups in a number of countries, and find it occurs gradually as the product cycle evolves and network effects develop. Together, the above evidence suggests product group growth is largely predictable and unrelated to the business cycle.

## 4 New Goods and the Consumer Price Index

### 4.1 Existing Evidence on CPI Bias: Overview

One prominent implication of variety growth is mismeasurement of the CPI. As already noted, the price of a unit of product group  $c_{jt}$  is

$$p_{jt} = A_t^{-\gamma} m_t^{-v-1+\frac{1}{\alpha}} \left( \int_0^{m_t} p_{j_{it}}^{\frac{\alpha}{\alpha-1}} di \right)^{\frac{\alpha-1}{\alpha}}, \quad (13)$$

where  $p_{j_{it}}$  denotes the price of brand  $c_{j_{it}}$  at time  $t$ .<sup>10</sup> Calculating the change in this ideal price index requires measuring the degree of brand and quality growth along with the attendant welfare gains. While in theory the welfare gain to a new good is the difference between its virtual price—the price at which demand is zero—and its actual price, incorporating such calculations for each good into a price index makes any systematic exercise intractable. According to [Abraham, Greenlees, and Moulton](#)

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<sup>10</sup>Calculating the CPI involves taking a expenditure-weighted average of such price indices for approximately 200 categories of goods. For most categories, the price index over individual goods in a category takes the form of a geometric average of prices; i.e.,  $p_{jt} = \prod_{i=1}^{m_t} (p_{j_{it}})^{\frac{1}{m_t}}$ . In contrast to my CES formulation, this implicitly assumes Cobb-Douglas utility over components. However, both are equal in a symmetric equilibrium and given this—as opposed to substitution effects—is my focus here, I retain the index corresponding to (1). Finally, as a practical matter, discussions of CPI bias refer to omissions from Eq (13); given the large but uncertain welfare gains to groups, their introduction would make the index highly imprecise and volatile.

(1998), the consumer surplus technique “may never be adaptable for implementation in a large, ongoing price measurement program like the CPI.” For this reason, the Bureau of Labor Statistics (BLS) omits the rise in consumer surplus associated with new products; i.e., they assume  $v = 0$ . Yet as stressed by Hausman (1997), this can be significant even for relatively similar goods such as new breakfast cereals. Using supermarket data, for example, Melser (2006) finds a new goods bias of between 1.2 and 2.4 percent a year.

Despite omitting the welfare gains associated with new brands, the BLS performs some quality adjustment. Yet Hausman (2002) describes such adjustments as “severely inadequate”. When a good becomes unavailable, the BLS must find a replacement item and assign any price differential to either quality growth or inflation; this represents an “item substitution.” According to Bils (2004), the BLS inaccurately attributes most price increases to inflation rather than quality growth; for example, Bils and Klenow (2001a) document an average inflation bias of 2.2 percent annually for durable goods.

Empirically, new goods play a large role in measured inflation. Covering the years 1983, 1984, and 1995, and 80 percent of U.S. consumer products, Moulton and Moses (1997) report item substitutions occurred for approximately 4 percent of price quotes each month in each of these years, but these goods accounted for 50 percent or more of CPI inflation each year. As Shapiro and Wilcox (1996) observe, “something quite dramatic on the pricing front happens when an old variety of an item disappears and a new one is introduced.”

#### 4.1.1 Decomposing the CPI Bias

In this section, I develop a time series estimate of the CPI bias. Imposing symmetry, the price index (13) reduces to

$$p_t A_t^{-\gamma} m_t^{-v},$$

where  $p_t$  denotes the price of each good at time  $t$ . Absent variety, this reduces to  $p_t$ ; i.e.,

the price of an average good. The true measure of annual inflation of product group  $j$ —and hence overall inflation—is thus approximately

$$\pi = g_p - \gamma g_A - v g_m,$$

where  $g_p$  denotes pure price inflation—unrelated to new goods— $g_A$  quality growth,  $g_m$  brand growth,  $\gamma$  the taste for quality, and  $v$  the taste for brands. Accounting for the fact the BLS does incorporate some quality adjustment, I decompose quality growth as

$$g_A = g_a + g'_a,$$

where  $g_a$  denotes the BLS estimate of quality growth, while  $g'_a$  captures omitted quality growth. [Moulton and Moses \(1997\)](#) report that the BLS estimated as much as 1 percent average quality growth in 1995, implying  $g_a \approx 1\%$ . By excluding the welfare gain to the introduction of a new good, the BLS implicitly set  $v$  to zero, implying an overall inflation bias of

$$\pi_{BIAS} = \gamma g'_a + v g_m. \tag{14}$$

The 1996 Boskin Commission Report estimated an average quality bias,  $\gamma g'_a$ , of .4 percentage points annually. In common with many studies, [Diewert \(1998\)](#) estimates an average annual new goods bias,  $v g_m$ , of .5 percentage points. This figure captures the consumer surplus arising from new goods. Combined, these common estimates imply an average upward bias of .9 percentage points per year.

## 4.2 Estimation of Bias

To derive a time series for  $\pi_{BIAS}$  in (14) and thus gauge the importance of variety growth for asset pricing, I derive estimates of the taste parameters,  $v$  and  $\gamma$ , together with brand and quality growth rates. Yet given the difficulties in quantifying variety and consumer surplus, the goal here is to provide a first-order approximation of the bias and its effect on consumption growth. I perform the exercise over the period 1951

to 2014. Restricting the sample to the post-war period avoids the data comparability issues in consumption data highlighted by [Romer \(1986\)](#). Given the usually high restocking of durable goods in the immediate post-war period, I start the estimation in 1951.

As is standard, I infer  $v$  from elasticities of demand across brands.<sup>11</sup> Corresponding to elasticities of demand between 3.5 and 6, [Rotemberg and Woodford \(1991\)](#) cite markups of between 20 and 40 percent. Based on these findings, I use an elasticity of 5 as a baseline, implying  $v \approx .3$ . Similar to [Hummels and Klenow \(2005\)](#), I set  $\gamma = 1$ , making proportional quality increases equivalent to proportional consumption increases.

### 4.3 Dynamic Goods and CPI Bias

Absent concrete data, I follow [Bils and Klenow \(2001b\)](#) and use the growth *dynamic* goods consumption to infer an estimate of variety growth. Available at annual and quarterly frequencies, dynamic goods are those exhibiting high item substitution rates, and hence most likely to incorporate new brands and quality improvements. In 1997, for example, 30 percent of goods required substitutions. Significantly, a high average substitution rate for a product category reflects greater product creation in that category; e.g., in 1997 the rate was .4 for tobacco products and 9.2 for household appliances ([Ekstrand, 1999](#)). Because they contain a large proportion of new goods, the consumption of dynamic goods should correlate with the welfare gains to expanding product variety and quality. For this reason, both [Bils and Klenow \(2001b\)](#) and [Nakamura and Steinsson \(2008\)](#) use substitution rates as a measure of product innovation. Most importantly, this proxy is contemporaneously correlated with variety growth; by contrast, with trademark or R & D data, there can be a lag between movement in the proxy and the realization of the bias.

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<sup>11</sup>According to the standard [Dixit and Stiglitz](#) model,  $c_{jt} = m_t^{(\frac{1}{\alpha}-1)} A_t^\gamma C_t$ , which corresponds to  $m_t^v A_t^\gamma C_t$  in my setup. For the purposes of the calibration, I set  $v = \frac{1}{\alpha} - 1$ , implying the elasticity of demand is  $\frac{1}{1-\alpha}$ .

To determine which goods are dynamic, I choose a cut-off substitution rate of one, implying dynamic goods comprise approximately 30 percent of consumption. Below this cut-off the categories comprise mainly services, which tend to be more stable in nature over time.<sup>12</sup> I use category substitution rates for the period 1998-2005, available from the appendix of Nakamura and Steinsson (2008) and reproduced in Table 8.<sup>13</sup> The dynamic basket comprises the following categories: transportation goods (e.g., cars), apparel, recreation goods (e.g., consumer electronics), household furnishing, processed foods, and unprocessed food.<sup>14</sup> To determine dynamic goods growth, I use the category's expenditure shares as a proportion of dynamic goods to calculate a weighted average of the growth rates of each category.

In calculating the rate of dynamic goods growth, an important issue here is the measurement of the consumption flow from consumer durables (henceforth *durables*). For this purpose, I assume the service flow from durables is proportional to the stock. Yet there are lags to the adjustment of durables, which are especially relevant in calculating the adjustment of consumption to stock returns in quarterly data. Recognizing this, I calculate the growth of the stock over a number of periods. Parker (2001) shows that the flow of durables cointegrates with the stock, and estimates the growth rate of the stock over a number of periods by taking the growth rate of the flow over that period. Exploiting this insight, I use the six-quarter growth rate of durable goods expenditure as an estimate of the growth rate of the durables stock over that period; for annual data, I use the two-year growth rate. In each case, I divide by the number of periods to estimate the growth rate in a given period. Most important, these time frames correspond

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<sup>12</sup>One exception is travel services. As a result of services related to new vehicle models, some goods within this category have high substitution rates; for example, goods comprising "automobile maintenance and repair" have a relatively high substitution rate. Yet, in the context of the model, these components more closely resemble inputs than final consumption goods. For this reason, I assume travel services represent static goods.

<sup>13</sup>Calculating the item substitution rate for a product involves dividing the number of product substitutions by the product's lifetime in the sample. For the *categories* listed, the substitution rate refers to the expenditure-weighted median monthly frequency of price changes associated with item substitutions within that category.

<sup>14</sup>Because items in a category typically have similar item substitution rates, a finer disaggregation would produce similar results.

to those yielding the best fits of the CCAPM in [Parker \(2001\)](#).

This metric constitutes a crude measure of brand *and* quality growth. Calculated over the period 1951-2014, the annual series has a mean growth rate of 2.5 percent and a standard deviation of 2.7 percent. Yet, to the extent the BLS does account for quality growth of approximately 1 percent, a more accurate measure of unaccounted for brand and quality growth is 1.5 percent. Given uncertainty about its decomposition, I attribute half of this to brand growth and half to quality growth. Taking the average of tastes for brand and quality, I infer an average annual bias of  $.3(.75) + 1(.75) \approx 1\%$ .

#### 4.4 Summary Statistics of Bias-Adjusted Series

Armed with the time series on bias, I construct a series for the CPI bias together with bias-adjusted series for both consumption growth and inflation. To calculate bias-adjusted inflation, I subtract the bias series from CPI inflation; to calculate bias-adjusted consumption growth, I add the bias to NIPA consumption growth. [Table 1](#) presents summary statistics for bias-adjusted measures of inflation and consumption. [Figure 3](#) shows the bias-adjusted series for inflation and consumption growth. While these are highly imperfect estimates, the figure clearly shows the lower average level of bias-adjusted inflation and the correspondingly higher rate of consumption growth. Particularly after 1980, the graph illustrates the greater volatility of bias-adjusted consumption growth. While not presented, the quarterly data exhibit a similar pattern.

In addition to presenting data on means and standard deviations, [Table 1](#) also presents data on the covariance of the bias-adjusted series with excess market returns. The excess market return is the real excess value-weighted return on all NYSE, AMEX, and NASDAQ stocks over the three-month Treasury bill, available from [Kenneth French's](#) website. Of particular relevance to asset pricing is the higher covariance of excess returns with bias-adjusted consumption growth; it is more than double that of the unadjusted series.

This aggregate evidence on the comovement of returns and variety growth is con-

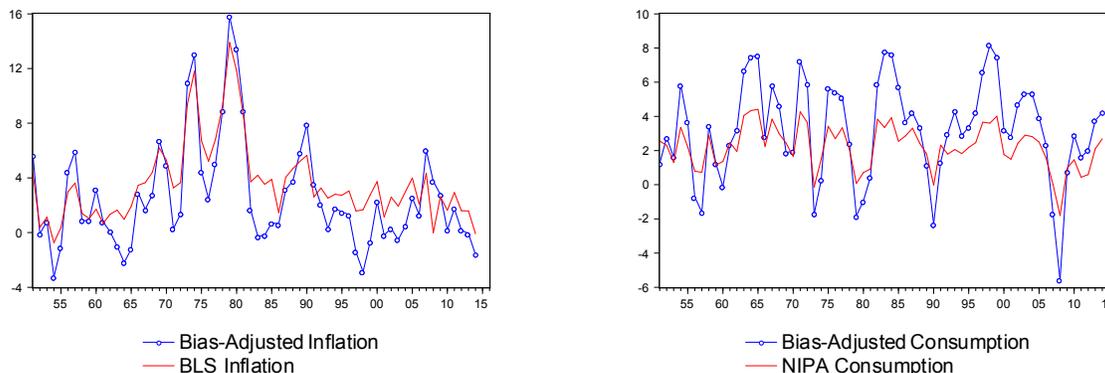


Figure 3: BIAS-ADJUSTED INFLATION AND CONSUMPTION GROWTH:U.S.,1951-2014  
 Source: BEA, BLS, and Author's Calculations.

Table 1: **Consumption and Inflation with Variety Adjustments, 1951-2014**

Stat	BLS Inf.	Bias-Adj Inf.	NIPA Cons.	Bias-Adj Cons.
Mean	3.57	2.59	.221	3.19
St. Dev	2.90	3.88	1.27	2.89
Corr(x, ep)	-0.41	-0.51	0.46	0.46
Cov(x, ep)	-0.22	-0.36	0.11	0.24
AR(1)	0.74	0.65	0.44	0.54

NOTE: This shows means and standard deviations of standard measures and bias-adjusted measures of inflation and consumption growth for the U.S over the period 1951-2014. The table also presents correlations and covariances of each measure with the equity premium.

sistent with studies at the micro level. At a higher level of disaggregation, Figure 4 depicts a close relationship between, for example, new food product introduction and excess returns. Reviewing new product announcements, [Chaney, Devinney, and Winer \(1991\)](#) find that a new product announcement raises the market value of a firm by an average of 26 million in 1972 dollars. This implies a positive relationship between a firm's variety growth and returns over time. Likewise, in an analysis of new product announcements between 1987-1995, [Chen, Ho, and Ik \(2005\)](#) find that the equity value gains of an innovating firm exceed the fall in returns amongst competitors over a two-day announcement period.

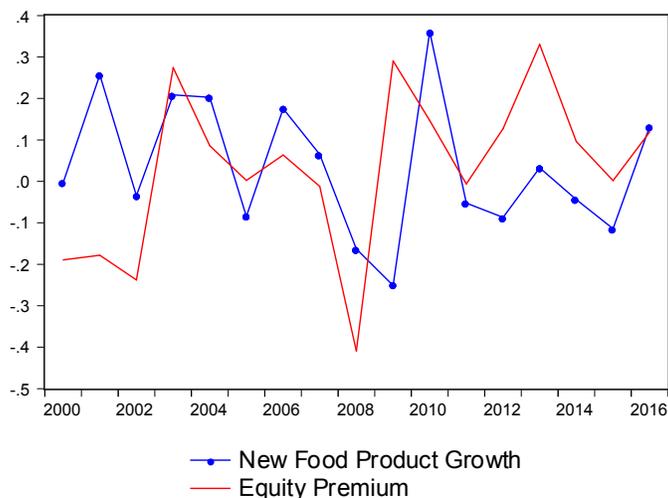


Figure 4: NEW FOOD PRODUCT GROWTH AND EQUITY PREMIUM: U.S., 2000-2016.  
SOURCE: MINTEL GNPD.

## 5 Calibration of Basic Model: Power Utility

The simple calculations presented in Section 4 show that variety growth raises the degree of consumption risk. To gauge the potential importance of new goods for asset values in a more systematic way, I now present a calibration encompassing a broad range of parameter values. The goal here is to provide an approximation of predicted returns in a standard benchmark setting with i.i.d. variables and power utility. In particular, I calibrate the expected log risk-free rate  $r_f$  from Eq. (11), and the log expected equity premium from Eq. (12). Given the uncertainty associated with the measurement of variety, I consider a range of parameters in the calibration.

### 5.1 Data and Parameter Values

Table 2 displays the baseline parameter values. All data are in annualized percentage terms. The data on nondurables and services consumption dates from 1929 to 2016 and is from the BEA. The rate of consumption growth is 2 percent with a standard deviation of 2 percent. The standard deviation of the equity premium is 20 percent

Table 2: **Baseline Parameters for Calibration: Power Utility**

DESCRIPTION	SYMBOL	VALUE	DESCRIPTION	SYMBOL	VALUE
Risk aversion	$\theta$	5.0	Mean brand growth	$\mu_m$	1.7
Rate of time preference	$\delta$	.01	Mean quality growth	$\mu_A$	0.4
Taste for brands	$v$	0.3	Mean group growth	$\mu_n$	1.7
Taste for quality	$\gamma$	1.0	Mean wage growth	$\mu_w$	2.0
Taste for groups	$\zeta$	6.9	Corr returns, cons growth	$\rho_{r,c}$	0.4
Std dev cons grow	$\sigma_c$	2.2	Corr returns, variety growth	$\rho_{r,A}, \rho_{r,m}$	0.5
Std dev brand growth	$\sigma_m$	2.2	Corr returns, group growth	$\rho_{r,n}$	0.0
Std dev quality growth	$\sigma_A$	1.0	Corr cons, variety growth	$\rho_{m,c}, \rho_{A,c}$	0.7
Std dev group growth	$\sigma_n$	0.0	Corr group, cons growth	$\rho_{n,c}$	0.0
Std dev equity returns	$\sigma_r$	20	Corr brand, quality growth	$\rho_{m,A}$	0.7
Mean cons growth	$c$	2.0	Corr group, variety growth	$\rho_{n,m}, \rho_{n,A}$	0.0

NOTE: Standard deviations and growth rates are in annualized percentage terms. The value of  $\zeta$  varies according to the parameter values chosen; this parameter derives from Eq. (10):  $\zeta = \frac{\theta\mu_c - \mu_w - v(1-\theta)\mu_m - \gamma(1-\theta)\mu_A}{\mu_n}$ .

over this period. I set wage growth equal to consumption growth, a stylized fact over long periods. For the baseline calibration, I set  $\theta = 5$ —a figure in line with standard measures of risk aversion, and also consistent with common empirical estimates of the intertemporal elasticity of substitution of aggregate consumption. For annual data, I set a rate of time preference of 1 percent.

Following [Greenwood and Uysal \(2005\)](#), I use trademark data to determine the growth and standard deviation of new goods growth. In an analysis of new product introductions advertised in the Wall Street Journal over the period 1975 – 1993, [Axarloglou \(2003\)](#) reports that the correlation between the number of new products issued and new trademark registration was .84. This finding suggests variation in new trademark registration reflects general trends in new product introduction. This would be true if the number of trademarked goods was roughly a constant fraction of all goods.<sup>15</sup>

Using this assumption, I use trademark data to estimate the growth rate of the stock

<sup>15</sup>To see this, suppose that the number of trademarked goods  $T$  is a constant fraction  $\kappa$  of the number of goods  $N$ ; i.e.,  $T = \kappa N$ . In this case, the correlation between changes in the number of trademarks and changes in the number of goods is one.

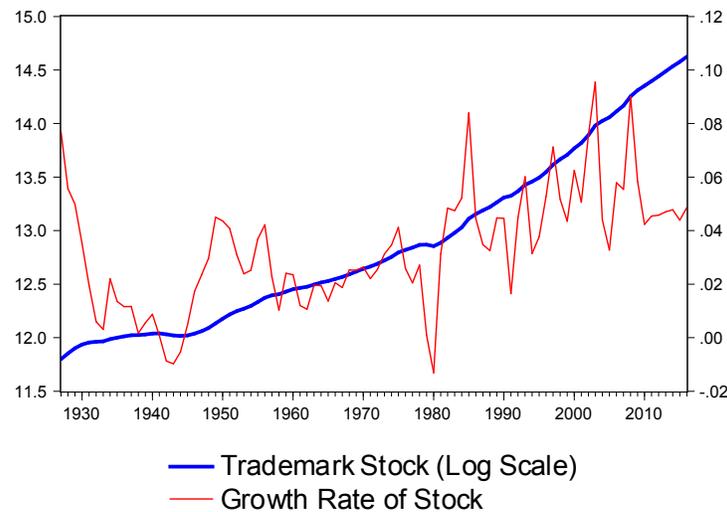


Figure 5: NEW TRADEMARKS REGISTERED IN UNITED STATES (LOG SCALE), 1927-2016.  
SOURCE: HISTORICAL STATISTICS OF THE UNITED STATES.

of goods, together with its standard deviation.<sup>16</sup> The trademark data is from the Historical Statistics of the United States over the ninety year period 1870-2016. Figure 5 shows the estimated stock and its growth rate over the period 1927-2016.

For concreteness, I assume there are  $n_t$  product groups at time  $t$ , and each group contains  $m_t$  brands; this results in  $n_t m_t$  goods. To estimate the growth of the stock of goods, I assume the number of goods is proportional to the stock of trademarks:  $T = \gamma n_t m_t$ . For small growth rates, this implies the mean growth of  $T$ ,  $\mu_T$ , satisfies  $\mu_T \approx \mu_n + \mu_m$ . Over the period 1927-2016, the annual growth in the stock of trademarks is 3.3 percent, while the standard deviation is 2.2 percent. Given no information on the decomposition of growth rates, I split growth evenly between groups and brands, implying  $\mu_m = \mu_n \approx 1.7\%$ . The fact product groups exhibit inelastic demand suggests little cyclical variation in their introduction. Furthermore, because firms rou-

<sup>16</sup>To estimate the stock of trademarks  $T$ , I use the perpetual inventory method. This implies the stock evolves according to the formula  $T_t = (1 - d)T_{t-1} + I_t + R_t$ , where  $d$  denotes the annual depreciation rate (the fraction of the stock that becomes unavailable commercially),  $I_t$  the number of new trademarks issued in year  $t$ , and  $R_t$  the number of trademark renewals in year  $t$ . I start the series from its initial stage in 1870, but to permit a sufficient burn-in period, and given many changes in trademark legislation in early years, I calculate growth rates from 1927. I take the depreciation rate from Landes and Posner (2003), who estimate an average depreciation rate for trademarks between 1934 and 1999 of 6.6 percent.

tinely announce the development of new product groups—such as drugs—years in advance, consumers often anticipate their release. Given this, I set the standard deviation of group growth to zero. Thus the estimate of brand growth has a mean of 1.7% and a standard deviation of 2.2%.

By contrast with brands, there is little concrete information on the variation of unaccounted for quality growth. But given the evidence of strongly procyclical variety growth in Section 3, and the fact the Boskin Commission estimate a mean growth rate of quality of .4 percent, a value of  $\sigma_A = 1$  percent seems a reasonable ballpark estimate. I consider a number of values in the calibration. Correlations of .7 between excess equity returns and each of brand and quality growth reflect the procyclical variation of these product categories.

Finally, to calibrate the parameter  $\zeta$ —which mediates the effect of groups on marginal utility—I use the stylized fact of stable labor hours over time, which from Eq. (10) implies  $\zeta = \frac{\theta\mu_c - \mu_w - v(1-\theta)\mu_m - \gamma(1-\theta)\mu_A}{\mu_n}$ . For the baseline parameters, this implies  $\zeta \approx 6.9$ .

## 5.2 Calibration of Expected Returns

Table 3 presents the calibrations of the annual expected risk-free rate and equity premium for the parameter values in each column. Column 1 corresponds to the standard CCAPM, which prices only expenditure risk. Column 2 presents predictions for the multi-good model and the baseline set of parameters. Incorporating changes in product variety causes the risk-free rate to fall from 10.4 to 1.6, and the equity premium to rise from .9 to 1.8. By making marginal utility more variable, changes in brand and quality growth make equities riskier, raising the equity premium. By slowing the decline in the marginal utility of consumption over time, product group growth raises savings and reduces the risk-free rate.

Columns 3 – 5 trace the effects of increasing the volatilities of brand and quality growth. By raising precautionary savings, these changes cause the risk-free rate to fall; by raising consumption risk, the equity premium increases. Comparing Column

Table 3: **Calibration of the Risk-Free Rate and Equity Premium**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\theta$	5	5	5	5	5	5	5	5	5	5	<b>3</b>	<b>7.5</b>
$v$	<b>0</b>	.3	.3	.3	.3	.3	.3	0	.6	.3	.3	.3
$\gamma$	<b>0</b>	1	1	1	1	1	1	1	0	<b>1.5</b>	1	1
$\sigma_{g_m}$	<b>0</b>	2	2	<b>3</b>	<b>3</b>	2	2	2	2	3	3	3
$\sigma_{g_A}$	<b>0</b>	1	<b>2</b>	1	<b>2</b>	1	1	1	1	1	1	1
$\sigma_{g_n}$	<b>0</b>	0	0	0	0	.5	<b>1.5</b>	<b>3</b>	0	0	0	0
$r_f$	<b>10.4</b>	1.6	1.0	1.5	.78	1.7	1.6	0.6	1.3	1.2	2.6	-0.2
<b>ep</b>	<b>0.9</b>	1.8	2.4	1.9	2.5	1.7	1.4	1.0	2.1	2.2	1.0	2.8

NOTE: The table presents the expected rates of return for the parameter values in each column.  $v$  denotes taste for brands and  $\gamma$  the taste for quality. The rate of time preference is  $\delta = .01$ . All figures are in annualized percentage terms. Except for Column (1), the expected log risk-free rate  $r_f$  is from Eq. (11), and the log expected equity premium is the left-hand-side of Eq. (12). Column (2) presents returns for the baseline parameters.

1 with Columns 3 – 5, it is apparent that risk to the composition of consumption comprises a significant part of consumption risk. Columns 6 – 8 show the effects of raising the variance of group growth. For these calibrations, I assume correlations of group growth and all other variables of 0.2. Given the assumed low correlation with returns, modest changes in variability has little effect on the results. However, raising volatility to 3 percent—i.e., to higher than that of brand or quality—almost reduces the equity premium to that of the standard CCAPM. Meanwhile, raising volatility has two effects on the risk-free rate. Through the standard channel, precautionary savings rises. Yet more variable group growth also acts as a hedge, attenuating changes in marginal utility. This reduces the need for savings. While the former effect dominates in Column 6 the latter effect dominates in Column 8.

Columns 9 and 10 show the effects of raising the taste for each of brands and quality. Because these changes make marginal utility more variable, they have the same qualitative effect as increasing the respective volatilities. In Column 9, doubling the taste for brands from .3 to .6 raises the equity premium by .3 percentage points. Meanwhile,

raising the taste for quality from 1 to 1.5 raises the premium by .4 percentage points. Columns 11 and 12 display the effects of changing the level of risk aversion. Raising the risk aversion to 7.5 leads to an equity premium that is often considered plausible.

Raising the correlation between equity returns and each of brand and quality growth has the same qualitative effects as raising the respective volatilities. Observing the expression for the equity premium in Eq.(12), a rise in the correlation between equity returns and brand growth by from .7 to 1 raises the equity premium by from 1.8 to 2 percent in the baseline case. A similar result holds in the case of quality growth. Thus, asset returns are relatively robust to these changes.

## 6 Long-Run Risk and Variety Growth

The calibration in Section 5 provides estimates for the levels of the risk-free rate and equity premium in a setting with i.i.d. processes for variety growth. Yet despite providing a useful benchmark for examining the level of returns, standard power utility has counterfactual implications for the *variation* in asset prices. With a coefficient of risk aversion above one, power utility implies an elasticity of intertemporal substitution (IES) below one. In turn, this implausibly implies higher expected growth and lower uncertainty each reduce the price-dividend ratio. Given conflicting evidence on the value of the IES, together with evidence of persistence I discuss below, in this section I incorporate new goods into a long-run risk setting with an IES exceeding one. This framework provides a natural setting for modelling variety growth. Especially over the long run, the number of products is path-dependent and contains a significant random element. One new product, for instance, could spur a series of derivative goods, and ultimately have a significant bearing on the number of products over time.

## 6.1 Model Assumptions

To isolate the effect of fluctuations in variety growth on asset prices, I make a number of assumptions. Throughout, I assume consumption growth is i.i.d. I set the rate of time preference to equal a risk-free rate of one percent. In contrast to the power utility setting where  $IES < 1$ , brands and consumption are now less substitutable within periods than consumption across periods; that is, there is a preference for intratemporal smoothing. As a result, brand growth in a period raises marginal utility and has the same qualitative effect as product groups. For ease of exposition, I therefore focus only on brands and set group growth to zero.<sup>17</sup>

Because the growth of trademarks is independent of consumption data and provides a natural and direct empirical counterpart to the concept of variety growth, I use trademarks as a proxy for brand growth to calibrate and simulate the model. As documented by [Broda and Weinstein \(2010\)](#), new goods incorporating quality improvements exhibit similar cyclical properties to brand growth. Recognizing this and the fact new brands often embody quality improvements, I assume the level of quality growth is proportional to brand growth; i.e.,  $g_A = kg_m$ . Noting the estimated average value for  $g_m$  of 1.7%, I choose  $k$  to match the annual quality bias of .4 percentage points reported by the Boskin Commission; this implies  $k \approx .2$ . Drawing on this result, I assume throughout that the level of quality bias takes the form  $A = m^2$ . Finally, I omit labour supply.

## 6.2 Evidence on Persistence in Variety Growth

To estimate the processes for variety and consumption, I use data from the ninety year period 1927-2016. While consumption growth has a first-order autocorrelation of .45 and a second-order value of .14 over this period, variety growth has respective values of .73 and .52. Reflecting this serial correlation, variance ratios for variety growth

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<sup>17</sup>Given the little volatility and acyclicity of groups, the ultimate effect of omitting them is the need to set a lower rate of time preference to match the risk-free rate.

rise from 1.77 at year two to 6.62 by year ten. By contrast, variance ratios for consumption reach 1.43 after two years, subsequently decline and reach .93 at year ten. Figure 2 provides some crude evidence of persistence. Commencing in 2008, the rate of new product introduction fell, and took eight years to revert to its previous 2008 level. Given the important role [Bansal and Yaron \(2004\)](#) ascribe to persistent trends, this evidence on variety growth points to a role for long-run risk.

In addition to persistence in the level of variety growth, there is also evidence of persistence in its volatility. In an AR(1) model of variety growth, the residuals follow a GARCH(1,1) process with a highly statistically significant GARCH coefficient of .65.<sup>18</sup> Figure 5 provides graphical evidence of time-variation in volatility. Measuring volatility at time  $t$  as the log of the sum of the absolute values of the residuals from the AR(3) model over the ensuing three periods, there is a negative relationship between the log price-earnings ratio and volatility over the period 1927 to 2016. The unconditional correlation of both series for the full time period is modest at -.09. Yet the strength of the relationship varies over time, and the series exhibits a correlation of -.50 up until 1990.<sup>19</sup> Taken together, this evidence points to a role for persistent volatility.

### 6.3 Long-Run Risk Model and Calibration

The following processes describe the dynamics of the model:

$$\begin{aligned}
 g_{c,t+1} &= \mu_c + \sigma_c \epsilon_{c,t+1} \\
 g_{m,t+1} &= \mu_m + x_t + \sigma_t \epsilon_{m,t+1} \\
 x_{t+1} &= \rho x_t + \phi_e \sigma_t \epsilon_{x,t+1} \\
 \sigma_{t+1}^2 &= \sigma^2 + v_1 (\sigma_t^2 - \sigma^2) + \sigma_v \epsilon_{\sigma,t+1} \\
 g_{d,t+1} &= \mu_d + \phi x_t + \psi_d \sigma_t \epsilon_{d,t+1},
 \end{aligned}$$

<sup>18</sup>The ARCH coefficient is .34 with a t-value of 1.59.

<sup>19</sup>According to the long-run risk model, the price-earnings ratio will also depend on the long-run trend in variety growth. As such, the strength of the relationship between the ratio and volatility will vary over time.

where the shock terms—of the form  $\epsilon_{z,t+1}$ —are all *i.i.d.*  $N(0,1)$ . Table 4 displays the simulation parameters, which are set at a monthly frequency. Following [Bansal and Yaron \(2004\)](#), I choose parameters of the variety processes above to match the annualized mean, variance, and autocorrelation of variety growth over the period 1927-2016. Based on evidence in Section 4, the annualized mean of brand growth is 1.7 percent. I set the volatility of the trend in expected variety growth to  $\phi_e = .07$ . I choose a rate of persistence  $\rho = .989$ , which is greater than the corresponding value [Bansal and Yaron \(2004\)](#) use for consumption, and matches the greater autocorrelation of variety growth.

Using aggregate dividends accruing to firms in the *S&P* index—available from [Goyal and Welch \(2008\)](#)—the correlation between annual variety and dividend growth is .2 over the period 1927-2016. Given this, and the role of product development for firm growth, I assume dividend growth depends on the trend in expected variety growth,  $x_t$ . I choose the parameter  $\mu_d$  to match the mean of dividend growth and  $\psi_d$  to match its standard deviation; setting  $\phi = 1.35$  ensures correlations between dividend and variety growth approximate those in the data.

The representative agent is infinitely lived and has utility

$$U_t = \left( \delta (c_{j_t})^{1-\frac{1}{\psi}} + (1-\delta) \left( \mathbb{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}},$$

where

$$c_{j_t} \equiv m_t^{v+1-\frac{1}{\alpha}} \left( \int_0^{m_t} (A_t c_{j_{it}})^\alpha di \right)^{\frac{1}{\alpha}}, \quad (15)$$

and  $\psi$  is the IES for the consumption bundle  $c_{j_t}$ ,  $\gamma$  is the coefficient of relative risk aversion, and  $\delta$  the rate of time preference. Imposing symmetry and substituting in the assumed relationship between brands and quality,  $A = m^2$ , this reduces to

$$U_t = \left( \delta (C_t m_t^v)^{1-\frac{1}{\psi}} + (1-\delta) \left( \mathbb{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}},$$

where  $C_t$  denotes real consumption expenditure. I set the coefficient of relative risk

aversion to 7.5—which is the lower value used by [Bansal and Yaron \(2004\)](#)—and the IES to 1.25. This parameter configuration implies a preference for the early resolution of uncertainty. To ensure the risk-free rate is 1 percent, I set  $\delta = .00001$ . With a taste for brands of .3, incorporating quality implies a value of  $v = 0.5$ .

## 6.4 Simulation of Long-Run Risk Model with Variety

Assuming joint log-normality of variety, consumption, and returns, I derive an analytical solution, and simulate the model at monthly frequency for a period of 100,000 months.<sup>20</sup> Table 5 presents the annualized results. Column 2 displays the results for the baseline parameters. The model predicts a relatively stable risk-free rate and an equity premium of 1.85 percent. Faced with a persistent negative shock to the expected growth trend or a rise in volatility, people shun risky financial assets and asset prices fall. At the same time, the changes raise the prospect of persistently lower and unstable welfare, and this raises marginal utility today. This confluence of low returns and higher marginal utility raises the expected equity premium. Examining the first row, the results are sensitive to changes in the love of variety parameter  $v$ ; raising  $v$  from .5 to .9 raises the equity premium by 2.37 percent.

The price-dividend ratio provides another way to evaluate the model. The standard deviation of the predicted equity premium and the log price-dividend ratios are lower than in the data. Over the period 1927-2016, the log annual price-dividend ratio has a first-order autocorrelation of .88 and a volatility of .47. In the benchmark simulation, the first-order autocorrelation of the log price-dividend ratio is .87, while the volatility is lower at .19. Tables 6 present results of regressions of excess returns, dividend growth and variety growth for holding periods of one, three, five, and seven years over the period 1927-2016. A high log price-dividend ratio predicts high variety growth and low excess returns, but has little predictive power for dividend growth.

To examine the model's implications for predictability, I perform 10,000 simulations,

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<sup>20</sup>The Appendix presents an analytical solution to the model.

Table 4: **Long-Run Risk Parameters: Monthly Frequency**

DESCRIPTION	SYMBOL	VALUE
STRUCTURAL PARAMETERS		
Mean consumption growth	$\mu_c$	0.00166
Consumption growth volatility	$\sigma_c$	0.006
Mean variety growth	$\mu_m$	0.00136
Long-run risk persistence	$\rho$	0.989
Long-run risk volatility multiple	$\phi_e$	0.07
Baseline volatility	$\sigma_m$	0.0035
Volatility of volatility	$\sigma_v$	0.0000018
Persistence of volatility	$v_1$	0.986
Mean dividend growth	$\mu_d$	0.0011
Dividend leverage	$\phi$	1.35
Dividend volatility multiple	$\psi_d$	7.7
PREFERENCE PARAMETERS		
Relative risk aversion	$\gamma$	7.5
Intertemporal Elasticity	$\psi$	1.25
Rate of time preference	$\delta$	0.00001
Taste for brands and quality	$v$	0.5

each 90 years in length—the length of the sample size—and average over simulation results; Table 7 presents the average slope coefficients and R-squared values. For baseline parameters, the simulated log price-dividend ratio predicts variety growth at all horizons. It is less successful in predicting excess returns. While the slope coefficient is negative at all horizons, and the R-squared value rises over time, it only reaches .06 at seven years. For dividend growth, the value reaches 14 percent at seven years.

Table 5: **Simulation Results from Long-Run Risk Model**

This table presents annualized simulation results for combinations of love of variety,  $v$ , and risk aversion  $\gamma$ .  $p - d$  denotes the log of the price-dividend ratio. Unless otherwise stated, risk aversion is  $\gamma = 7.5$  and  $v = .5$ . Throughout, the rate of time preference,  $\delta > 0$ , is set to match a risk-free rate of 1%. Column 2 presents results for the benchmark parameters. The data covers the period 1927-2016.

	Data	$v = .5$	$v = .3$	$v = .7$	$v = .9$	$\gamma = 5$	$\gamma = 10$
$\mathbb{E}(r - r_f)$	5.93	1.85	1.01	2.88	4.22	1.01	3.07
$\sigma(r - r_f)$	20.0	13.09	12.77	13.60	14.32	12.94	13.41
$\sigma(r_f)$	2.86	0.06	0.04	0.09	.11	0.06	0.06
$p - d$	3.53	3.63	4.66	2.51	1.29	4.59	2.61
$\sigma(p - d)$	0.47	0.17	0.17	0.19	.20	0.17	0.18
$AC1(\exp(p - d))$	0.88	0.87	0.88	0.87	.87	0.88	0.87

Table 6: Predictive Regressions: Data: U.S., 1927-2016

Years ( $k$ )	EXCESS RETURNS			DIVIDENDS			VARIETY		
	Slope	$t$ -stat	$R^2$	Slope	$t$ -stat	$R^2$	Slope	$t$ -stat	$R^2$
1	-0.08	-1.83	0.04	0.06	2.42	0.06	0.02	4.88	0.21
3	-0.23	-3.15	0.10	0.08	1.75	0.03	0.07	6.31	0.32
5	-0.37	-4.34	0.18	0.07	1.40	0.02	0.12	7.09	0.37
7	-0.48	-5.82	0.29	0.06	1.21	0.02	0.16	7.21	0.39

NOTE: This table presents results from predictive regressions of the form  $X_{t,t+k} = a + b(p_t - d_t) + \epsilon_{t+k}$  where  $X_{t,t+k}$  is the cumulate log change in the variable  $X$  from time  $t$  to  $t + k$ . Excess returns refer to the real excess value-weighted returns on all NYSE, AMEX, and NASDAQ stocks over the three-month Treasury bill, available from Kenneth French's website. Dividend data is from [Goyal and Welch \(2008\)](#). The estimated trademark stock proxies for variety.

Table 7: Predictive Regressions: Simulations for Benchmark Parameters

Years	EXCESS RETURNS		DIVIDENDS		VARIETY	
	Slope	$R^2$	Slope	$R^2$	Slope	$R^2$
1	-0.05	0.02	0.12	0.06	0.12	0.08
3	-0.11	0.04	0.28	0.11	0.28	0.20
5	-0.17	0.05	0.38	0.13	0.38	0.28
7	-0.20	0.06	0.45	0.14	0.45	0.30

NOTE: The reported results are the average slope regression coefficients and R-squared values of 10,000 simulations, each with a length of ninety years.

## 7 Cross-Sectional Return Variation

In this section, I examine whether variety growth can explain the spread of cross-sectional returns. To test the model, I use the proxies for variety growth—dynamic goods growth and trademark growth—developed in Sections 4 and 5.

### 7.1 Data and Methodology

I use 30 test assets, comprising 25 Fama-French portfolios sorted by book-to-market ratio together with five industry portfolios. These exhibit a large spread in returns, while the inclusion of industry portfolios mitigates the critique of ?. Data on all portfolios comes from Kenneth French's website. Given the relatively large number of assets together with a relatively short time series, I use the Fama-MacBeth procedure. In the first stage, I calculate the betas from a time series regression covering the whole time period. Given these are estimated variables, I present Shanken-corrected standard errors throughout.

The test is of the power utility model of Section, which—assuming stable group growth—implies a two-factor asset pricing model. According to the model, both consumption and variety growth should be significant, with the intercept insignificant. For annual data, the regressions are from 1951-2014; for quarterly data they are from 1951Q1-2015Q2. Although data is available until 2016Q2, there are the maximum respective time periods available given the forward-looking nature of the dynamic goods proxy.

### 7.2 Cross-Sectional Results

Table 9 summarizes the Fama-MacBeth results for annual data. Column 1 illustrates the poor performance of the CCAPM, where the  $R$ -squared is .18. Column 2 displays results for the variety-adjusted model, where dynamic goods proxy for variety. Here the  $R$ -squared is .44. Consistent with the model, the intercept is insignificant while

dynamic goods are significant. Consumption, however, is insignificant; one reason for this is multicollinearity between consumption and dynamics good growth. The cross-sectional price of variety risk is positive. A one unit standard deviation increase in the variety beta leads to a 2.31 percentage point rise in expected return. I include dynamic goods as a single variable in Column 3. Comparing the R-squared values in the first three columns, most of the explanatory power comes from the presence of dynamic goods. In Column 4, I use trademark stock growth as a proxy for variety growth. Available only at annual frequency, this independent measure of variety is highly significant. Comparing columns, this proxy performs best on goodness-of-fit with an R-squared of .61. A one standard deviation increase in the variety beta leads to a rise in expected returns of 2.8 percentage points. While not reported, the portfolio-based Fama-French model has an  $R^2$  of .67. The price of variety risk is broadly consistent across regressions. As a robustness check, Table 10 summarizes the results for quarterly data over the period 1951Q1-2015Q2. The results broadly parallel those for quarterly data.

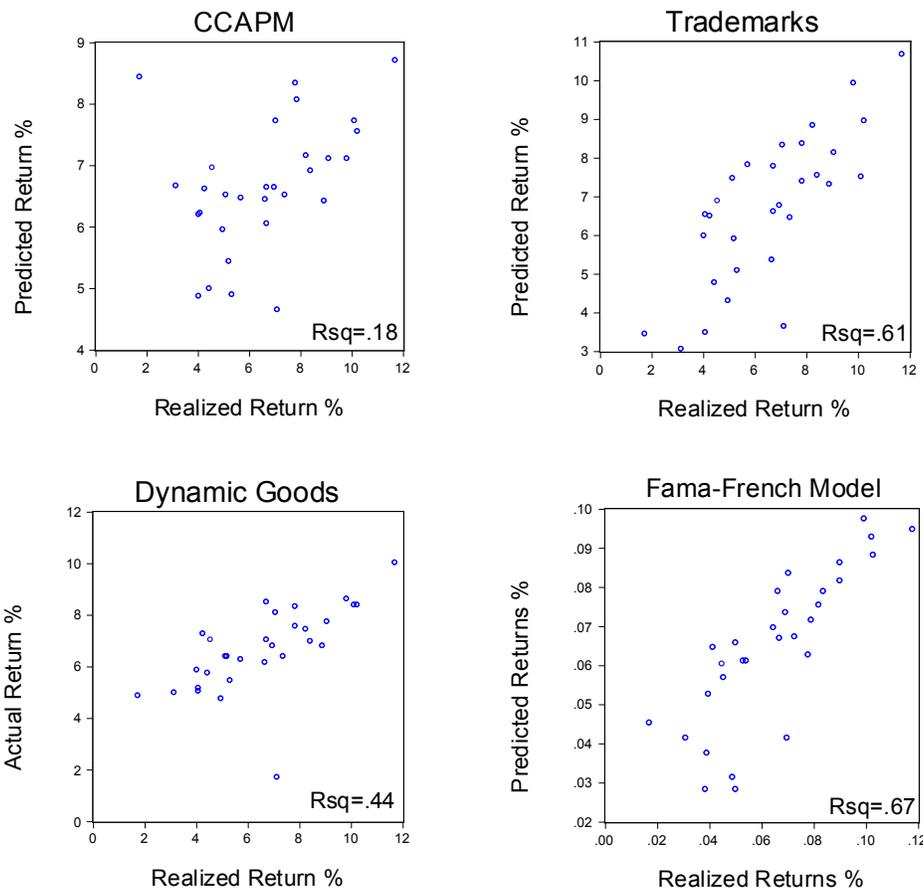


Figure 6: Fitted and Realized Excess Returns: Annual Data, 1951-2014. This shows realized versus predicted excess returns for the CCAPM, the two-factor extensions of the CCAPM models (trademarks and dynamic goods, resp.) and the Fama-French model. Dots represent excess returns on 5 industry portfolios and 25 Fama-French portfolios sorted by size and book-to-market ratio.

### 7.3 Variety Growth and Industry Returns

Given the procyclicality of variety growth and the positive relationship between returns and new product introduction, the model predicts that firms who introduce more products command higher average returns. Reviewing product introductions [Chaney, Devinney, and Winer \(1991\)](#) finds evidence of this. To examine this further, I identify the most innovative consumption-oriented industries among the Fama-French 17 in-

dustry portfolios. Table 11 shows the average annual returns for these industries and the remaining ones over the period 1930-2016. The high item substitution industries exhibit higher average returns of 1.3%, and the difference in means between both categories is statistically significant.

Further cross-sectional evidence comes indirectly from examining durability. According to Table 8, there is a positive relationship between an industry's substitution rate and the durability of its output.<sup>21</sup> Using NAICS input-output tables to isolate each firm's contribution to final output—thereby controlling for spillovers across industries—Gomes, Kogan, and Yogo (2009) sort firms into three groups: consumer durables, nondurables, and services. They find a strong positive relationship between the degree of durability and average returns. While they attribute this to the greater volatility of durables demand, the framework here provides another interpretation of their results: the more durable the output, the more innovative the products, and hence the greater the variety risk.

Finally, I test the model using the 17 Fama-French industry portfolios. One advantage of this approach is variety betas implicitly account for interactions across industries. As shown in Table 12, however, the relationship is weak. Yet one issue here is the degree of product innovation in industries can change over time, making betas unstable. Another issue is the small spread of returns across industries; the portfolio-based Fama-French model also performs poorly on industry portfolios. To improve identification, I consider a study by Bils and Klenow (2001b) who analyze variety growth over the period 1959q1-1999q4, and report higher variety growth in dynamic industries over the period 1979q1-1999q4. This would lead to a sharper distinction between static and dynamic industries over the latter period. The more pronounced differentials in variety growth would lead to sharper predictions of the model over this period. Consistent with a role for variety in explaining returns, the model has an  $R^2$  of 37 percent from 1979q1-1999q4, but only 11 percent over the earlier period.

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<sup>21</sup> As already noted, clothing is an exception. Because of seasonal introductions, the high substitution rate overestimates the degree of product innovation.

## 8 Conclusion

In a dynamic economy, product innovation provides a natural link between consumption and stock returns. While firms devote significant resources to developing new products, consumers devote significant resources to purchasing them. Recognizing this, this paper incorporates new goods into the CCAPM and long-run risk models, and examines the relationship between expanding product variety and asset prices.

Over the business cycle, fluctuating brand and quality growth amplify consumption risk and raise the equity premium. Over the long run, product group growth raises the incentive to save and reduces the risk-free rate. In each model, the introduction of new goods attenuates the equity premium and risk-free rate puzzles. In a long-run risk setting, fluctuations in variety growth generate a time-varying risk premium.

A limitation of the analysis is the lack of data on variety growth and consumer surplus. Relying on proxies for these intangible variables makes the findings more suggestive than concrete. Nonetheless, three factors suggest new goods play a role in asset pricing. First, introducing new goods into different frameworks and calibrating the models for a broad range of parameter values shows the basic idea is robust. Second, the proxies for variety growth—while crude—perform better than consumption growth in explaining the cross-sectional variation in returns. More generally, new goods comprise a large share of the consumption of upper income groups—making them especially relevant for the marginal investor.

Overall, by taking a broader view of consumption, the framework highlights another dimension of consumption risk, and complements existing consumption-based models. Because the concept of a good becomes increasingly intangible as economies advance, the issues raised here will likely become more relevant over time.

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Table 8: **Substitution Rates and CPI Weights for Different Industries, 1998-2005**

Item	Substitution Rate	CPI Weight
Vehicle Fuel	0.2	5.1
Utilities	0.6	5.3
Services (excl. Travel)	0.9	38.5
Other goods	1.0	5.4
Unprocessed Food	1.2	5.9
Processed Food	1.3	8.2
Travel	1.9	5.5
Household Furnishing	5.0	5.9
Recreation Goods	6.3	3.6
Apparel	9.9	6.5
Transportation Goods	10.2	8.3

This table is from [Nakamura and Steinsson \(2008\)](#) and presents industry substitution rates together with the corresponding weights in the CPI.

Table 9: **Fama-MacBeth Annual Regressions**

	(1)	(2)	(3)	(4)	(5)
Factor	CCAPM	DYN-CCAPM	DYN	TM-CCAPM	TM
Variety		1.12 (1.67)	1.12 (1.87)	1.79 (2.67)	1.41 (2.39)
Consum.	0.63 (1.43)	0.35 (0.87)		0.81 (1.98)	
Intercept	1.89 (0.74)	2.32 (0.89)	0.87 (0.34)	0.13 (2.24)	7.15 (9.37)
RMSE	1.46	1.20	1.28	1.01	1.40
$R^2$	.18	.44	.38	.61	.25

This table reports results from cross-sectional Fama-MacBeth regressions for various consumption-based models over the period 1951-2014. The test assets are the 25 Fama-French portfolios sorted by size and book-to-market ratio at annual frequency and 5 industry portfolios. Returns refer to real excess returns on these portfolios over 3-month Treasury bills. The left-hand column lists the risk factors. *Variety* refers to variety growth; *Consum.* refers to nondurables and services consumption growth. The top row identifies the consumption-based model. *DYN-CCAPM* is a two-factor model, where dynamic goods proxy for variety; with *TM-CCAPM*, trademarks proxy for variety. *DYN* refers to the one-factor model where dynamic goods proxy for variety. *TM* refers to the model where trademark growth proxies for variety growth. The first two rows report the factor risk premia estimated from a cross-sectional regression of average returns on estimated betas. The third row reports the intercept. *RMSE* refers to the root-mean-square error of the regression. *t*-values are calculated using Shanken standard errors with a three-lag [Newey and West \(1987\)](#) correction for autocorrelation and heteroskedasticity. *t*-values are in parentheses.

Table 10: **Fama-MacBeth Quarterly Regressions**

	(1)	(2)	(3)
Factor	CCAPM	DYN-CCAPM	DYN
Variety		0.75 (3.07)	0.57 (2.13)
Consum.	0.11 (0.59)	-0.13 (-0.83)	
Intercept	0.98 (0.75)	-0.16 (-0.21)	-0.31 (-0.86)
<i>RMSE</i>	0.18	0.13	.16
$R^2$	0.02	0.48	0.24

This table reports results from cross-sectional Fama-MacBeth regressions for various consumption-based models over the period 1951Q1-2015Q2. The test assets are the 25 Fama-French portfolios sorted by size and book-to-market ratio at annual frequency and 5 industry portfolios. Returns refer to real excess returns on these portfolios over 3-month Treasury bills. The left-hand column lists the risk factors. *Variety* refers to variety growth; *Consum.* refers to nondurables and services consumption growth. The top row identifies the consumption-based model. *DYN-CCAPM* is a two-factor model, where dynamic goods proxy for variety. *DYN* refers to the one-factor model where dynamic goods proxy for variety. The first two rows report the factor risk premia estimated from a cross-sectional regression of average returns on estimated betas. The third row reports the intercept. *RMSE* refers to the root-mean-square error of the regression. *t*-values are calculated using Shanken standard errors with a three-lag [Newey and West \(1987\)](#) correction for autocorrelation and heteroskedasticity. *t*-values are in parentheses.

**Table 11: Average Returns Across Fama-French 17 Industry Portfolios, 1930-2012**

High Variety Sector	Return	Low Variety Sector	Return
Food	9.50	Mines	8.73
Clothing	9.63	Oil	9.69
Consumer Durables	9.31	Chemicals	10.01
Cars	12.19	Consumables	9.74
Retail	10.08	Construction	9.36
		Steel	8.31
		Fabricated Products	8.90
		Machinery	10.40
		Transport	8.82
		Utilities	8.13
		Financial	9.85
		Other	8.36
Average	10.14		9.19

The table reports average annual returns over the period 1930-2016 for the Fama-French 17 industry portfolios. The left-hand column reports returns for consumption-oriented sectors with high product introduction rates or exposed to variety risk. The category “consumables” comprises drugs, perfume, tobacco, and soap.

Table 12: **Test of VCCAPM Using Industry Portfolios, 1951-2012**

	(1)	(2)	(3)
Period	1951 Q1-2015 Q2	1959 Q1-1978 Q4	1979 Q1-1999 Q4
Variety	0.08 (0.67)	0.1366 (0.80)	.38 (2.01)
Consum.	-0.05 (-0.56)	0.05 (0.37)	0.11 (0.58)
Intercept	1.00 (3.33)	-0.63 (-1.43)	0.69 (1.15)
$R^2$	0.19	0.11	0.37

Column 1 presents the output from Fama-MacBeth regressions of the excess returns on 17 industry returns on variety and consumption growth in quarterly data over the period 1951Q1-2015Q2. *Variety* refers to variety growth and *Consum.* to nondurables and services consumption growth. Columns 2 and 3 show the results of the same regression over the periods 1959 Q1-1978 Q4 and 1979 Q1-1999 Q4, respectively. The latter period was marked by relatively high variety growth in certain sectors (Bils and Klenow, 2001b). *t*-statistics are in parenthesis.